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Distributions of the fertilizer production function in relation to decision-making

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DISTRIBUTIONS OF THE FERTILIZER PRODUCTION FUNCTION IN RELATION TO DECISION-MAKING

by

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DISTRIBUTIONS OF THE FERTILIZER PRODUCTION FUNCTION IN RELATION TO DECISION-MAKING

INTRODUCTION

The production of any product requires time. Regardless of the nature of the product, its production is not instantaneous, nor is it produced without its production having first been planned. Hence, plans must be made at one point in time for the production of the product at some future date. Since time is involved and the demand for any product changes with time, the entrepreneur must make production plans in the face of uncertainty with respect to either the price at which the total production can be sold, or the quantity which can be sold at a given price.

In addition to the market uncertainty discussed above, producers of agricultural commodities, crops especially, must make plans in the face of variability and uncertainty as to the outcome of technological processes. Farmers in general are more dependent upon the vagrancies of weather as to the outcome of their production processes[1] than any other group of producers. On the other hand, due to the operation of government price support programs, farmers as a group are faced with less market uncertainty than many other groups of producers. The farmer producing the major crops knows within fairly narrow limits, at least when annual production plans are made, the price at which the product can be sold. Since this is true, it will be assumed throughout this study that prices are known with certainty.

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[1]As used in this study a production process will imply all factors of production fixed except weather.

As noted above, the outcome of most crop production processes depends upon weather conditions. With our present state of knowledge we are not able to control weather to any appreciable extent, or even predict it for any considerable period ahead. Therefore, the producer of crops must continue to make production plans in the face of technical uncertainty. He does not know at the time production plans are laid whether use of 45 pounds of nitrogen per acre on corn will increase yield over no nitrogen by 4 bushels or by 40 bushels. Hence, he must formulate some notion of the possible outcomes and their frequency of occurrence in order to make intelligent production plans. In addition, the results of following various systems of planning for a period of years would be helpful.

In this study an attempt is made to estimate a sample of the nitrogen production functions for corn from the distribution of such functions. The functions estimated are then used to estimate a sample of the possible responses from the application of specified levels of nitrogen. A comparison of the results from following several planning procedures which may be used in planning the application of nitrogen to corn is also presented. Because of limitations of the data, however, this study is considered to be of an exploratory nature. It is not a study upon which recommendations to farmers will be based.

REVIEW OF LITERATURE

Many investigators have estimated production functions for different agricultural situations. In most of these studies, however, the production functions have been estimated for a single year or for an average of several years. Heady[1] estimated production functions for different types of Iowa farms for the year 1939. In this study he used total production as the dependent variable and land, months of labor, equipment, livestock and feed, and miscellaneous operating expense as the independent variables. All variables were measured in dollar terms except labor. Tintner[2] estimated a production function for American agriculture using annual data for the period 1920-1941. His variables were volume of agricultural production, employment in agriculture, agricultural operating capital and time.

Research workers have long realized that climatic factors affect crop yields. Moreover, they have long realized that weather conditions play an important part in determining response to various fertilizers. Baker and Vandecaveye stated in 1935 that:

> It is a common experience in field experiments to find that repeated applications of the same fertilizers to the same soil do not give the same results from year to year. Variations in available moisture and especially a deficiency of moisture are generally accepted as the major responsible factors.[3]

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[1]Earl O. Heady. Production functions from a random sample. Journal of Farm Economics. 28: 989-1004. 1946.

[2]G. Tintner. An application of the variate difference method to multiple regression. Econometrica. 12: 97-113. 1944.

[3]G. O. Baker and S. C. Vandecaveye. Climatic factors affecting yield of oats and vetch. Journal of Agricultural Research. 50: 967. 1935.

In a North Carolina Experiment Station Bulletin Krantz made the following statement in explaining the difference in responses to the same level of applied nitrogen: "The degree of response to applied nitrogen was influenced mainly by climatic conditions and the level of nitrogen in the soil."[1] In the same report he presented the following data concerning corn yields and nitrogen applied. Under "fair to good" conditions yields obtained from 0, 40, 80, and 120 pounds of nitrogen were 29, 55, 76, and 88 bushels, respectively. For the same rates of nitrogen yields under "dry" conditions were 27, 45, 53, and 56 bushels, respectively.[2] By presentation of the data in this manner the author implies that there are two nitrogen production functions for corn. One is realized under "dry" conditions and the other under "fair to good" conditions.

It was pointed out by Heady that the agricultural producer is faced with many production functions. He states, "The farm producer is not faced with a single production function. Instead, he is faced with an entire distribution of production functions."[3] Characterization of such a distribution from empirical data is attempted in the present study.

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[1]E. A. Krantz. Fertilize corn for higher yields. North Carolina Agricultural Experiment Station Bulletin 366. 1949. p. 10.

[2]Ibid. p. 12.

[3]Earl O. Heady. Economics of agricultural production and resource use. New York, Prentice-Hall, Inc. 1952. p. 444.

LIMITATIONS OF DATA AND PROCEDURES

Data

Data for the study were obtained from Agronomy Department files and publications through courtesy of members of the Agronomy Department, Mississippi Agricultural Experiment Station. These data were the per acre yields of corn obtained on experimental plots at the Delta Branch Experiment Station at Stoneville, Mississippi from 1921-1952. Nitrogen application on these plots ranged from 0-45 pounds per acre in 7½ pound increments (Table 1).

As is usually the case when secondary data are used, the data are not ideal for the purpose at hand. In many years of the period the upper limit of the range of nitrogen applied was not great enough so that marginal product had begun to decrease. In other years it was decreasing only very slowly at the highest rate of application. Hence, the usefulness of the data for economic analysis is severely limited.

A production function for nitrogen for any given year would be of the form

$$Y = f(X_1 | X_2 | X_3, X_4 \cdots X_n) \tag{1}$$

where Y refers to corn yields, X_1 refers to nitrogen applied, X_2 refers to weather conditions, and X_3, X_4, $\cdots$ X_n refers to the other factors of production. The vertical bar to the right of X_1 in this equation indicates that all factors of production other than nitrogen applied are held constant within years. The one to the right of X_2 specifies that all factors of production other than nitrogen applied and weather conditions are held constant between years.

Table 1. Corn yields obtained from various rates of nitrogen, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952[a]

Year	Nitrogen applied per acre						
	None	$7\frac{1}{2}$ lb.	15 lb.	$22\frac{1}{2}$ lb.	30 lb.	$37\frac{1}{2}$ lb.	45 lb.
				(bushels)			
1921	40.5	41.4	43.2	44.5	46.0	44.8	44.0
1922	32.7	34.4	38.4	45.1	48.2	54.8	56.4
1923	30.3	31.7	35.1	42.9	44.2	43.7	50.4
1924	26.3	29.5	32.3	39.2	40.2	42.3	42.4
1925	26.1	29.1	31.2	32.5	30.4	29.5	28.9
1926	26.3	30.2	34.9	39.2	43.2	47.5	46.0
1927	30.9	36.4	46.0	50.3	62.8	62.5	61.7
1928	31.6	40.3	49.3	55.9	66.5	73.9	73.7
1929	12.6	14.5	21.1	22.8	30.9	32.6	33.4
1930	44.0	46.9	52.0	55.3	57.5	59.6	60.5
1931	45.3	62.1	63.8	59.7	55.5	52.5	55.7
1932	14.3	15.4	21.7	25.1	35.3	38.1	39.9
1933	15.3	14.2	16.5	18.4	22.4	25.9	28.4
1934	12.4	14.2	19.3	23.0	27.0	28.8	28.9
1935	11.5	12.2	18.1	22.4	29.9	35.8	36.5
1936	10.0	12.8	13.5	15.2	19.0	21.1	20.4
1937	11.1	16.1	20.2	27.5	33.4	33.7	35.9
1938	14.5	15.4	21.4	23.3	32.5	40.5	44.3
1939	11.2	14.8	17.7	21.8	25.9	28.7	30.5
1940	47.9	49.3	52.9	57.9	61.4	63.8	67.2
1941	30.9	32.4	39.2	41.7	43.5	45.8	50.3
1942	29.1	34.4	43.2	47.4	54.3	57.9	62.5
1943	15.4	20.1	25.2	30.6	34.6	33.9	38.9
1944	15.6	15.7	16.9	23.1	24.3	23.9	22.1
1945	43.2	48.9	44.8	65.6	67.1	74.6	77.8
1946	39.2	39.6	44.9	55.1	62.3	67.3	72.1
1947	20.1	24.0	28.8	32.6	39.1	43.8	48.5
1948	22.9	23.7	31.7	33.6	41.3	44.8	48.1
1949	35.8	40.0	45.4	54.4	62.5	68.5	74.0
1940	27.7	30.6	39.6	46.5	50.4	59.8	63.7
1951	22.5	23.2	27.9	32.8	33.8	38.5	45.5
1952	21.4	25.2	30.9	34.2	41.6	49.1	51.2

[a]Perrin H. Grissom. Corn fertilization in the Yazoo - Mississippi Delta. Mississippi Agricultural Experiment Station Bulletin 454. Data for 1948-1952 from Agronomy Department files.

A comparison of the functions for the individual years is valid only if the factors to the right of X_2 were actually held constant. Many changes of a minor nature in the quality or quantity of factors of production other than nitrogen have been made since the experiment was begun. There were a few changes such as changes in variety in 1945 and afterwards which might be termed major changes. However, none of the trends in yields for the 32-year period for the various levels of nitrogen were significantly different from zero at the 5 percent level of probability (Table 2). Moreover, they were not significantly different from each other at the same level of probability.

Table 2. Comparison of trends in corn yields for different rates of nitrogen, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Rates of N[a]	d.f.	Sx^2	Sxy	Sy^2	$Sy^2 - \frac{(Sxy)^2}{Sx^2}$	d.f.	M.S.	r[b]
0	31	2,728	-130.6	3,817.14	3,810.89	30		-.040
1	31	2,728	-127.8	4,214.35	4,208.36	30		-.038
2	31	2,728	53.9	4,817.85	4,816.79	30		.015
3	31	2,728	212.6	5,799.72	5,783.15	30		.053
4	31	2,728	347.7	6,155.72	6,111.40	30		.085
5	31	2,728	715.1	7,225.90	7,038.45	30		.161
6	31	2,728	1,116.5	8,040.49	7,583.54	30		.238
					39,352.58	210	187.39	
Sums	217	19,096	2,187.4	40,071.17	39,820.61	216		
Difference for testing $H_o: \beta_o = \beta_1 = \cdots = \beta_6$					468.03	6	78.00	

$F = 78.00/187.39 = .416$

[a]Rates of nitrogen in 7½ pound increments per acre.

[b]Values of r greater than .349 would indicate trends significant at the 5 percent level of probability.

The observed value of F indicates that the trends are more nearly alike than would be obtained one-half the time by sampling from the same population. Hence, it was thought that, for the purpose at hand, all factors of production to the right of X_2 in equation (1) could be assumed to have been constant between years.

Selecting Functions

Scatter diagrams of yield plotted against rates of nitrogen were prepared as the first step in selecting the type functions to fit to the annual data. These graphs indicated that the functions should be such as to allow constant, increasing, or decreasing marginal productivity, or a combination of the latter two. For some years it appeared that the function should permit decreasing marginal productivity with the marginal product becoming negative[1]

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[1] At first thought negative marginal productivity of nitrogen may not seem logical with the highest rate of nitrogen used in the experiment being 45 pounds per acre. However, there are factors which lend support to the hypothesis that such actually was the case.

If negative marginal productivity were indicated for only one or two years of the entire period, the indication might have resulted from random fluctuations. But, negative marginal productivity was indicated for 12 of the 32 years. See Table B in the Appendix.

In the experiment a rotation of cotton-corn-oats was practiced with each plot receiving the same rate of nitrogen every year regardless of the crop planted. This procedure resulted in the same plots being planted to corn every third year. If the indications of negative marginal productivity occurred every third year, it might have been as a result of a part or all of the plots receiving the higher rates of nitrogen having lower yielding ability, and the procedure used in estimating marginal productivity. However, such was not the case. The years in which negative marginal productivity was indicated were 1921, 1924-1927, 1929, 1930, 1932, 1934, 1936, 1937, and 1944. If negative marginal productivity did in fact occur, a possible explanation is the interaction of weather conditions and other factors of production in the years concerned. Rainfall averaged 2.84, 2.52, and 2.33 inches for June, July, and August during these years compared to 3.92, 4.48, and 2.88 inches during the balance of the period. Temperature averaged 80.6, 83.0, and 82.3 degrees Fahrenheit for June, July, and August during the twelve years compared to 79.0, 81.4, and 81.3 degrees for the other twenty years. See Table C in the Appendix.

within the range of observations. Functions which would allow increasing and decreasing marginal productivity with the marginal product becoming negative within the range of observations were indicated for other years.

In order to allow the flexibility needed to describe the relationships mathematically, the following type functions were fitted to the data for each year:[1]

1. $Y = a + bX$ (2)
2. $Y = a + bX + cX^2$ (3)
3. $Y = a + bX^2 + cX^3$ (4)
4. $Y = a + bX + cX^{\frac{1}{2}}$ (5)
5. $Y = a + bX^2 + cX^{\frac{1}{2}}$ (6)

In many cases two or more of these different type functions did a satisfactory job of describing the relationships over the range of observations. It was necessary, therefore, to use some sort of criterion to determine which of the different type functions did the best job of characterizing the relationship for each year. Size of the mean square of the deviations from regression was used as this criterion. The type function yielding the smallest mean square of the deviations from regression was the type selected as best describing the relationship for each year individually. Further calculations were made under the assumption that the function selected is the true function for the year in question.

There are probably many other type functions which would describe the relationships equally as well as, or perhaps even better than, the types actually fitted. However, the types fitted allow the total product to

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[1] In these functions Y denotes corn yields per acre and X denotes nitrogen applied per acre.

behave in a logical fashion over the range involved, and each of the functions selected for the individual years does an excellent job of describing the relationship for that year.

Fitting Functions

In the experimental set-up there were always more plots on which no nitrogen was applied than plots on which the different rates of nitrogen were applied.[1] Since the functions were fitted to the means of all plots receiving the same treatment for a given year, the problem of homoscedasticity was encountered. To meet this problem weighted regression can be used. However, since scatter diagrams indicated that excellent fits would be obtained, ordinary regression techniques were used to simplify calculations.[2]

Use of ordinary regression procedures to calculate b and c in equations (2)-(6) leads to the solution of the following equation and systems of equations for each of the 32 years.[3]

1. $Y = a + bX$

$$b = \frac{Sx_1y}{Sx_1^2} \tag{7}$$

2. $Y = a + bX + cX^2$

$$\begin{aligned} bSx_1^2 + cSx_1x_2 &= Sx_1y \\ bSx_1x_2 + cSx_2^2 &= Sx_2y \end{aligned} \tag{8}$$

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[1] Rates of nitrogen were assigned to the different plots in a systematic fashion, i.e., 0, 7½, 15, 22½, 0, 30, 37½, 45, 0, 7½, etc.

[2] Other things being equal the greater the correlation coefficient the smaller will be the differences obtained using the two procedures.

[3] In the actual fitting of the functions Y denotes corn yields in bushels per acre and X denotes the application of nitrogen per acre in 7½ pound increments. For clarity in the normal equations, X is coded as X_1, X^2 as X_2, X^3 as X_3, and X^4 as X_4.

3. $Y = a + bX^2 + cX^3$

$$bSx_2^2 + cSx_2x_3 = Sx_2y$$
$$bSx_2x_3 + cSx_3^2 = Sx_3y \qquad (9)$$

4. $Y = a + bX + cX^{\frac{1}{2}}$

$$bSx_1^2 + cSx_1x_4 = Sx_1y$$
$$bSx_1x_4 + cSx_4^2 = Sx_4y \qquad (10)$$

5. $Y = a + bX^2 + cX^{\frac{1}{2}}$

$$bSx_2^2 + cSx_2x_4 = Sx_2y$$
$$bSx_2x_4 + cSx_4^2 = Sx_4y \qquad (11)$$

A regression plane fitted by least squares procedures always passes through the means of all variables. Therefore, a may be calculated for each of the different type functions in the order listed from the following equations.[1]

1. $a = \bar{Y} - b\bar{X}$ (12)
2. $a = \bar{Y} - b\bar{X}_1 - c\bar{X}_2$ (13)
3. $a = \bar{Y} - b\bar{X}_2 - c\bar{X}_3$ (14)
4. $a = \bar{Y} - b\bar{X}_1 - c\bar{X}_4$ (15)
5. $a = \bar{Y} - b\bar{X}_2 - c\bar{X}_4$ (16)

The functions of primary interest were the response to nitrogen functions rather than the production functions as such. Transition to response to nitrogen functions was accomplished by subtracting the constant a in each of the production functions selected for the individual years from that function. This placed all the annual functions on a directly comparable basis--all response functions have the value zero when X equals zero.

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[1] $\bar{Y}$ denotes the mean of Y, $\bar{X}_1$ the mean of X_1, etc.

For reasons to be pointed out later, it was desired to estimate an average of the annual response functions. This average response function was estimated by (1) evaluating each of the annual response functions at each level of nitrogen actually used in the experiment, (2) calculating the mean response for each level of nitrogen from the figures obtained in (1) above, and (3) fitting each of the different type functions indicated in equations (2)-(6) to the mean response for each level of nitrogen. A first quartile response function, a median response function, and a third quartile response function was fitted using the same principle. In each case the type function yielding the smallest mean square of the deviations from regression was chosen as the type function giving the best fit.

In fitting functions to the mean, first quartile, median, and third quartile responses, as determined from the annual response functions, weighted regression techniques were used. Weights were determined by dividing a constant by estimates of the variance of responses at the different levels of nitrogen. This was deemed advisable because of extremely wide differences in the estimated variances at the different levels of nitrogen.

Use of weighted regression procedures in calculating $\underline{b}$ and $\underline{c}$ for the different type functions indicated in equations (2)-(6) leads to the solution of the same equation and systems of equations, equations (7)-(11), as does use of ordinary regression methods. However, the sums of squares and sums of cross products in these equations must now be interpreted as designating weighted sums of squares and weighted sums of cross products. Solution of these equations yield weighted regression coefficients.

Calculation of the constant $\underline{a}$ for each of the different type functions fitted to the mean, the first quartile, the median, and the third quartile responses follows the same principle as before. The best fit passes through the weighted means of all variables. Therefore, equations (12)-(16) may be used to calculate these constants if the means of the variables involved are interpreted as weighted means.

RESULTS AND ECONOMIC INTERPRETATION

Fitting Functions

Correlation coefficients obtained as a result of fitting the five type functions specified in equations (2)-(6) to the annual data indicate that either of the different type functions would have been statistically acceptable in most instances (Table 3). Correlation coefficients for each of the different type functions were all significant at the 1 percent level of probability for each of the 32 years except for 1921, 1925, and 1944. For two of these three years there was at least one function which resulted in a correlation coefficient significant at the 1 percent level of probability.

Two of the correlation coefficients for 1921 were significant at the 1 percent level of probability, two were significant at the 5 percent level, and the other approached significance at the 5 percent level. For 1925 three were significant at the 5 percent level and for 1944 one was significant at the 1 percent level and three were significant at the 5 percent level.

Functions Selected

As stated previously, the function selected as best describing the relationship for each of the years was the function yielding the smallest mean square of the deviations from regression. This procedure resulted in the type function $Y = a + bX$ for the three years 1923, 1941, and 1950; the type $Y = a + bX + cX^2$ for the seven years 1928, 1931, 1939, 1942, 1943, 1945 and 1951; and for all other years except 1925, 1933, and 1947 (a total of 19 years) the type $Y = a + bX^2 + cX^3$. $Y = a + bX + cX^{\frac{1}{2}}$ was the type function which gave the best fit for 1933 and 1947, and $Y = a + bX^2 + cX^{\frac{1}{2}}$ the type for 1925.

Table 3. Results of fitting various type functions to the corn-nitrogen data, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1921						
1	41.3	.718		14.429	1.652	.797
2	40.0	2.368	-.275	20.781	.477	.957
3	40.8	.739	-.1097	21.671	.254	.977
4	40.2	-.308	2.760	17.256	1.358	.872
5	39.9	-.057	2.777	18.036	1.163	.892
1922						
1	31.3	4.346		528.960	2.186	.990
2	31.6	3.968	.063	529.295	2.649	.990
3	32.8	1.827	-.1948	536.282	.902	.997
4	32.4	5.363	-2.737	531.741	2.037	.992
5	31.6	.435	4.528	520.185	4.926	.982
1923						
1	29.8	3.336		311.556	4.584	.965
2	29.4	3.764	-.071	311.984	5.623	.966
3	31.0	1.432	-.1561	307.705	6.693	.959
4	30.0	3.477	-.379	311.609	5.717	.965
5	29.4	.274	4.442	305.328	7.287	.955
1924						
1	27.2	2.921		238.973	3.916	.961
2	25.4	5.121	-.367	250.266	2.072	.984
3	27.3	1.786	-.2394	250.950	1.901	.985
4	25.8	1.656	3.405	243.277	3.819	.970
5	25.2	.097	6.149	239.508	4.762	.962

[a]Functions of the type $Y = a + bX$, $Y = a + bX + cX^2$, $Y = a + bX^2 + cX^3$, $Y = a + bX + cX^{\frac{1}{2}}$, and $Y = a + bX^{\frac{1}{2}} + cX^{\frac{1}{2}}$ are denoted by 1, 2, 3, 4, and 5, respectively. In these functions Y denotes total yield of corn in bushels per acre and X denotes increments of 7½ pounds of nitrogen per acre.

[b]Correlation coefficients greater than .754 and .874 denote significance at the 5 percent and 1 percent probability levels, respectively, for functions of the first type. For the other type functions correlation coefficients greater than .881 and .949 denote significance at the 5 percent and 1 percent levels, respectively.

Table 3. (Continued)

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1925						
1	28.8	.300		2.520	4.411	.032
2	26.4	3.143	-.474	21.378	.799	.934
3	28.1	.648	-.1078	12.630	2.986	.717
4	25.8	-2.255	6.875	20.066	1.127	.904
5	25.9	-.213	4.229	21.737	.709	.941
1926						
1	27.3	3.643		371.571	3.652	.976
2	25.5	5.757	-.352	382.002	1.957	.990
3	27.6	2.091	-.2630	385.149	1.170	.994
4	25.9	2.428	3.269	375.539	3.572	.981
5	25.2	.166	6.984	369.619	5.052	.974
1927						
1	32.8	5.764		930.356	18.487	.954
2	29.0	10.379	-.769	980.036	10.608	.979
3	32.6	3.684	-.4823	998.663	6.032	.988
4	29.8	3.201	6.899	948.025	18.691	.963
5	28.7	.177	12.355	932.670	22.530	.955
1928						
1	33.3	7.525		1,585.518	9.018	.986
2	30.7	10.704	-.530	1,609.092	5.379	.993
3	34.7	3.938	-.4763	1,606.829	5.945	.993
4	31.0	5.558	5.293	1,595.918	8.673	.989
5	29.9	.419	13.255	1,574.846	13.941	.983
1929						
1	12.4	3.871		419.663	4.065	.977
2	11.4	5.100	-.205	423.185	4.201	.981
3	13.0	2.022	-.2434	431.010	2.245	.990
4	12.1	3.627	.656	419.823	5.042	.977
5	11.4	.267	5.937	409.800	7.547	.965
1930						
1	11.4	2.918		238.389	1.613	.984
2	10.8	3.682	-.127	239.752	1.676	.986
3	11.9	1.519	-.1823	246.351	.026	.9998
4	11.5	2.983	-.174	238.400	6.514	.984
5	10.9	.219	4.171	231.592	3.716	.969

Table 3. (Continued)

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1931						
1	45.1	2.871		230.863	2.081	.978
2	43.5	4.814	-.324	239.671	.400	.997
3	45.4	1.613	-.2019	233.688	1.895	.984
4	43.6	1.580	3.476	235.349	1.480	.988
5	43.2	.104	5.950	232.569	2.175	.982
1932						
1	12.5	4.850		658.630	5.468	.980
2	12.5	4.879	-.005	658.632	6.834	.980
3	13.8	2.254	-.2546	677.222	2.187	.994
4	13.8	5.926	-2.897	661.745	6.056	.982
5	12.8	.463	5.376	642.325	10.911	.968
1933						
1	12.8	2.450		168.070	2.685	.962
2	14.5	.436	.336	177.537	.990	.989
3	14.4	.654	-.0425	179.072	.606	.993
4	15.4	4.603	-5.793	180.527	.242	.997
5	14.8	.395	.150	177.156	1.085	.988
1934						
1	12.6	3.086		266.606	2.702	.976
2	11.2	4.829	-.290	273.693	1.606	.988
3	12.9	1.787	-.2253	278.187	.482	.997
4	11.8	2.356	1.964	268.038	3.020	.978
5	11.2	.155	5.640	261.951	4.542	.967
1935						
1	9.4	4.786		641.286	4.302	.984
2	9.8	4.329	.076	641.773	5.255	.984
3	10.9	2.110	-.2310	658.892	.976	.997
4	11.1	6.223	-3.869	646.842	3.988	.988
5	10.1	.495	4.699	628.121	8.668	.973
1936						
1	10.3	1.904		101.460	1.208	.972
2	9.9	2.354	-.075	101.933	1.392	.974
3	10.7	.957	-.1130	103.767	.933	.982
4	10.0	1.651	.679	101.631	1.467	.972
5	9.7	.133	2.935	100.419	1.770	.967

Table 3. (Continued)

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1937						
1	12.2	4.386		538.566	5.401	.976
2	10.1	6.943	-.426	553.823	2.937	.990
3	12.7	2.484	-.3110	554.199	2.842	.990
4	10.5	2.915	3.959	544.384	5.296	.981
5	9.8	.198	8.423	535.815	7.438	.973
1938						
1	11.3	5.382		811.089	7.072	.979
2	13.6	2.589	.465	829.289	4.290	.990
3	14.2	1.688	-.1391	835.713	2.684	.994
4	14.6	8.246	-7.708	833.144	3.326	.992
5	13.7	.713	2.865	824.777	5.418	.987
1939						
1	11.4	3.354		314.900	.562	.996
2	10.9	4.032	-.113	315.975	.434	.997
3	12.4	1.554	-.1768	314.632	.769	.995
4	11.0	3.015	.910	315.208	.625	.996
5	10.6	.243	5.013	311.352	1.589	.990
1940						
1	47.0	3.407		325.041	.808	.994
2	47.0	3.336	.012	325.053	1.007	.994
3	48.2	1.437	-.1540	326.627	.613	.996
4	47.6	3.979	-1.540	325.921	.790	.995
5	47.0	.321	3.873	319.229	2.463	.985
1941						
1	30.9	3.189		284.803	1.843	.984
2	30.4	3.825	-.106	285.746	2.068	.986
3	32.2	1.332	-.1434	276.405	4.403	.970
4	30.5	2.846	.923	285.119	2.224	.985
5	30.1	.231	4.779	281.857	3.040	.979
1942						
1	30.0	5.654		894.960	2.231	.994
2	28.6	7.375	-.287	901.875	1.060	.998
3	31.8	2.634	-.3015	882.687	5.857	.987
4	28.7	4.517	3.059	898.434	1.920	.996
5	28.0	.360	9.271	888.680	4.358	.990

Table 3. (Continued)

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1943						
1	16.9	3.839		412.723	3.837	.978
2	15.1	6.004	-.361	423.653	2.064	.990
3	17.8	1.975	-.2383	407.231	4.170	.971
4	15.0	2.233	4.324	419.663	3.062	.986
5	14.5	.163	7.592	415.732	4.044	.981
1944						
1	15.6	1.546		66.960	5.571	.840
2	13.9	3.511	-.327	75.963	4.713	.895
3	14.8	1.344	-.1910	88.735	1.520	.967
4	14.9	.963	1.570	67.875	6.735	.846
5	14.3	.019	3.671	65.718	7.274	.833
1945						
1	44.1	5.946		990.080	4.127	.990
2	42.5	7.811	-.311	998.190	3.132	.989
3	45.8	2.820	-.3256	981.034	7.421	.985
4	42.8	4.834	2.993	993.405	4.328	.991
5	41.9	.379	9.714	980.938	7.445	.985
1946						
1	36.0	6.125		1,050.438	6.496	.985
2	36.9	5.089	.173	1,052.940	7.494	.986
3	38.2	2.563	-.2714	1,076.515	1.600	.997
4	38.7	8.423	-6.183	1,064.630	4.572	.992
5	37.4	.673	5.361	1,031.916	12.750	.976
1947						
1	19.3	4.825		651.858	.496	.998
2	19.8	4.189	.106	652.800	.384	.999
3	21.6	1.798	-.1769	648.735	1.400	.996
4	20.1	5.482	-1.768	653.018	.330	.999
5	19.5	.472	5.285	648.795	1.386	.996
1948						
1	21.6	4.550		579.670	2.889	.988
2	21.6	4.436	.019	579.700	3.604	.988
3	23.2	1.933	-.2080	583.918	2.550	.991
4	22.5	5.392	-2.266	581.575	3.136	.989
5	21.7	.433	5.096	568.733	6.346	.978

Table 3. (Continued)

Year and type function[a]	a	b	c	Reduction due to regression	Mean square deviations from regression	Correlation coefficient[b]
1949						
1	34.2	6.739		1,271.703	2.238	.996
2	34.6	6.189	.092	1,272.409	2.621	.996
3	36.8	2.753	-.2886	1,279.671	.806	.999
4	35.6	7.951	-3.261	1,275.652	1.810	.997
5	34.5	.657	7.345	1,255.153	6.935	.989
1950						
1	26.5	6.329		1,121.423	2.594	.994
2	26.6	6.257	.012	1,121.435	3.240	.994
3	29.0	2.588	-.2724	1,117.071	4.331	.992
4	27.3	7.033	-1.894	1,122.755	2.910	.995
5	26.3	.578	7.527	1,105.637	7.189	.987
1951						
1	20.7	3.768		397.509	3.073	.981
2	22.1	2.075	.282	404.196	2.170	.989
3	23.2	1.033	-.0729	400.652	3.056	.985
4	22.5	5.300	-4.122	403.818	2.264	.989
5	22.0	.467	2.556	402.827	2.512	.988
1952						
1	20.4	5.282		781.229	2.293	.993
2	20.9	4.661	.104	782.130	2.641	.993
3	22.5	2.083	-.2130	784.252	2.110	.995
4	21.3	6.132	-2.287	783.170	2.381	.994
5	20.6	.519	5.723	774.966	4.432	.989

The equations for 1933 and 1947 both yielded an area of negative marginal productivity first, then an area of increasing positive marginal productivity. This does not meet the test of logic. However, the minimum total product (the numerically largest negative marginal productivity) would occur at the level of .026 increments[1] of nitrogen for 1947. Hence, for all practical purposes the function shows increasing marginal productivity throughout the range of observations and was used to describe the relationship for this year. For 1933 the minimum total product would occur at the level of .396 increments[2] of nitrogen. This would mean that application of nitrogen would decrease total yield until $7\frac{1}{2}$ (.396) or 2.97 pounds of nitrogen were applied. This does not seem logical. Therefore, the type function yielding the second smallest mean square of the deviations from regression was selected as the function to characterize the relationship for 1933. Thus, the function of type $Y = a + bX^2 + cX^3$ was used for all succeeding calculations.

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[1]The function for 1947 is $Y = 20.1 + 5.482X - 1.7682X^{\frac{1}{2}}$ where Y denotes bushels of corn per acre and X nitrogen applied per acre in $7\frac{1}{2}$ pound increments. Total product is a minimum where $\frac{dy}{dx} = 0$. $\frac{dy}{dx} = 5.4821 - .8841X^{-\frac{1}{2}} = 0$. $X = .026$.

[2]The function which yielded the smallest mean square for 1933 was $Y = 15.4 + 4.603X - 5.7928X^{\frac{1}{2}}$ where X and Y have the same meaning as in the footnote directly above. Total product would be a minimum where $\frac{dy}{dx} = 0$. $\frac{dy}{dx} = 4.603 + 2.8964X^{-\frac{1}{2}} = 0$. $X = .396$.

Table 4. Annual corn response to nitrogen functions,[a] Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Function: Y =	Mean square	Correlation coefficient
1921	$.739X^2 - .1097X^3$	.254	.977[b]
1922	$1.827X^2 - .1948X^3$	.902	.997
1923	$3.336X$	4.584	.965[c]
1924	$1.786X^2 - .2304X^3$	1.901	.985
1925	$-.213X^2 + 4.229X^2$	.709	.941
1926	$2.091X^2 - .2630X^3$	1.170	.994
1927	$3.684X^2 - .4823X^3$	6.032	.988
1928	$10.704X - .530X^2$	5.379	.993
1929	$2.022X^2 - .2434X^3$	2.245	.990
1930	$1.519X^2 - .1823X^3$	.026	.9998
1931	$4.814X - .324X^2$	.400	.997
1932	$2.254X^2 - .2546X^3$	2.187	.994
1933	$.654X^2 - .0425X^3$	.606	.993
1934	$1.787X^2 - .2253X^3$	.482	.997
1935	$2.110X^2 - .2310X^3$	.976	.997
1936	$.957X^2 - .1130X^3$	.933	.982
1937	$2.484X^2 - .3110X^3$	2.842	.990
1938	$1.688X^2 - .1391X^3$	2.684	.994
1939	$4.032X - .113X^2$	.434	.997
1940	$1.437X^2 - .1540X^3$	.613	.996
1941	$3.189X$	1.843	.984
1942	$7.375X - .287X^2$	1.060	.998
1943	$6.004X - .361X^2$	2.064	.990
1944	$1.344X^2 - .1910X^3$	1.520	.967
1945	$7.811X - .311X^2$	3.132	.989
1946	$2.563X^2 - .2714X^3$	1.600	.997
1947	$5.482X - 1.768X^2$	.330	.999
1948	$1.933X^2 - .2080X^3$	2.550	.991
1949	$2.753X^2 - .2886X^3$	.806	.999
1950	$6.329X$	2.594	.994
1951	$2.075X + .282X^2$	2.170	.989
1952	$2.083X^2 - .2130X^3$	2.110	.995

[a] Y denotes response in corn yields in bushels per acre and X denotes pounds of nitrogen applied in 7½ pound increments.

[b] A correlation coefficient greater than .949 is significant at the 1 percent level of probability for all curvilinear functions.

[c] A correlation coefficient greater than .874 is significant at the 1 percent level of probability for linear functions.

The Sample of Functions

The functions chosen for the individual years, together with the resulting correlation coefficients, are presented in Table 4 as response to nitrogen functions.[1] It will be noted from the size of the correlation coefficients that the equations do an excellent job of describing mathematically the relationships between responses in corn yields and nitrogen applied over the range of observation.

It was assumed earlier that a comparison of the annual response functions is a valid comparison. If it can be assumed also that weather conditions are random in nature, then together the annual functions can be viewed as a random sample from the distribution of response functions. Heady[2] states "Meteorologists have not been able to establish definite weather cycles in parts of the Cornbelt and Southeast, and speculate that each year's weather must be viewed 'randomly and independently'." Halcrow[3] states:

> Thus changes in weather from year-to-year usually have been regarded as random fluctuations and scepticism is general as to the existence of periodic cycles in meteorological data. This does not imply that the irregular fluctuations are entirely accidental or fortuitous, but for purposes of prediction the pattern and sequence of weather phenomena take on the character of a random statistical probability.

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[1] By subtracting the constant a from the production functions in Table 3 a function is obtained which shows the total response due to nitrogen at the various levels of application.

[2] Heady. Economics of agricultural production and resource use. p. 481.

[3] Harold G. Halcrow. Problem of farm business survival - discussion. Journal of Farm Economics. 31: 951. 1949.

Shepherd[1] states:

> Various investigators in the past have thought that they had discovered cycles in the weather--three-year cycles, eight-year cycles, eleven-year cycles, etc. These periods are so varied and the cycles are so poorly defined, that there is a real question whether there are really any cycles at all.

So the assumption probably is not too far from the facts of the situation.

Uncertainty in Production

An evaluation of the annual response functions at the levels of nitrogen used in the experiment shows wide variation in possible outcomes for each nitrogen level (Table 5). If the annual response functions can be viewed as a random sample of response functions, then values of these functions at the various levels of nitrogen can be viewed as random samples of the possible responses from the application of given levels of nitrogen. Values of the annual response functions at the different levels of nitrogen are shown in array in Table 6. These data indicate that as the rate of nitrogen is increased, uncertainty as to the responses which will be obtained increases also.[2]

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[1]Geoffrey S. Shepherd. Agricultural price analysis. 3rd edition. Ames, Iowa. The Iowa State College Press. 1950. pp. 41-42.

[2]Using the estimated variances as a measure of uncertainty, the relative uncertainty is shown in Table E in the Appendix. This increase in uncertainty as the rate of nitrogen is increased is probably a partial explanation as to why farmers in general consistently apply less than the recommended rates of fertilizers.

Table 5. Responses to various levels of nitrogen,[a] Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Increments of $7\frac{1}{2}$ pounds of nitrogen					
	1	2	3	4	5	6
			(bushels)			
1921	.6	2.1	3.7	4.8	4.8	2.9
1922	1.6	5.7	11.1	16.7	21.3	23.7
1923	3.3	6.7	10.0	13.3	16.7	20.0
1924	1.6	5.3	9.9	13.9	15.8	14.6
1925	4.0	5.1	5.4	5.1	4.1	2.6
1926	1.8	6.3	11.7	16.7	19.4	18.5
1927	3.2	10.8	20.2	28.1	31.8	28.5
1928	10.2	19.3	27.3	34.3	40.3	45.1
1929	1.8	6.2	11.6	16.7	21.0	20.2
1930	1.3	4.6	8.8	12.6	15.2	15.2
1931	4.5	8.3	11.5	14.1	16.0	17.3
1932	2.0	7.0	13.4	19.7	24.5	26.1
1933	.7	2.3	4.8	7.8	11.1	14.4
1934	1.6	5.3	10.0	14.2	16.5	15.6
1935	1.9	6.6	12.8	19.0	23.9	26.1
1936	.9	2.9	5.5	8.1	9.8	10.0
1937	2.1	7.4	13.9	19.8	23.2	22.2
1938	1.6	5.7	11.4	18.1	24.8	30.8
1939	3.9	7.6	11.1	14.3	17.4	20.1
1940	1.2	4.5	8.7	13.2	16.7	18.5
1941	3.2	6.4	9.6	12.8	15.9	19.1
1942	7.1	13.6	19.5	24.9	29.7	33.9
1943	5.6	10.6	14.8	18.2	21.0	23.0
1944	1.1	3.9	6.9	9.3	9.7	7.1
1945	7.5	14.4	20.6	26.2	31.3	35.7
1946	2.3	8.1	15.8	23.6	30.2	33.7
1947	3.7	8.5	13.4	18.4	23.5	28.6
1948	1.7	6.0	11.8	17.6	22.3	24.7
1949	2.5	8.7	17.0	25.5	32.7	36.8
1950	6.3	12.7	19.0	25.3	31.6	38.0
1951	2.4	5.2	8.7	12.8	17.4	22.6
1952	1.9	6.6	12.9	19.7	25.5	29.0

[a]Calculated from response functions in Table 4.

Table 6. Array of responses to various levels of nitrogen, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Type average or quartile	Increments of 7½ pounds of nitrogen					
	1	2	3	4	5	6
	(bushels)					
	.6	2.1	3.7	4.8	4.1	2.6
	.7	2.3	4.8	5.1	4.8	2.9
	.9	2.9	5.4	7.8	9.7	7.1
	1.1	3.9	5.5	8.1	9.8	10.0
	1.2	4.5	6.9	9.3	11.1	14.4
	1.3	4.6	8.7	12.6	15.2	14.6
	1.6	5.1	8.7	12.8	15.8	15.2
	1.6	5.2	8.8	12.8	15.9	15.6
	1.6	5.3	9.6	13.2	16.0	17.3
	1.6	5.3	9.9	13.3	16.5	18.5
	1.7	5.7	10.0	13.9	16.7	18.5
	1.8	5.7	10.0	14.1	16.7	19.1
	1.8	6.0	11.1	14.2	17.4	20.0
	1.9	6.2	11.1	14.3	17.4	20.1
	1.9	6.3	11.4	16.7	19.4	20.2
	2.0	6.4	11.5	16.7	20.1	22.2
	2.1	6.6	11.6	16.7	21.0	22.6
	2.3	6.6	11.7	17.6	21.3	23.0
	2.4	6.7	11.8	18.1	22.3	23.7
	2.5	7.0	12.8	18.2	23.2	24.7
	3.2	7.4	12.9	18.4	23.5	26.1
	3.2	7.6	13.4	19.0	23.9	26.1
	3.3	8.1	13.4	19.7	24.5	28.5
	3.7	8.3	13.9	19.7	24.8	28.6
	3.9	8.5	14.8	19.8	25.5	29.0
	4.0	8.7	15.8	23.6	29.7	30.8
	4.5	10.6	17.0	24.9	30.2	33.7
	5.6	10.8	19.0	25.3	31.3	33.9
	6.3	12.7	19.5	25.5	31.6	35.7
	7.1	13.6	20.2	26.2	31.8	36.8
	7.5	14.4	20.6	28.1	32.7	38.0
	10.2	19.3	27.3	34.3	40.3	45.1
Mean	2.97	7.32	12.28	17.02	20.76	22.64
Median	2.05	6.50	11.55	16.70	20.55	22.40
First quartile	1.60	5.22	9.00	12.90	15.92	16.78
Third quartile	3.85	8.45	14.58	19.78	25.32	28.90

Planning Production

Production is carried on in most instances for the sake of profit.[1] Hence, the quantity of any factor of production which should be used is that amount which maximizes profits. This amount depends upon the productivity of the factor and price of the factor and price of the product.

Hicks[2] lists three conditions which must be fulfilled before a firm engaged in production is in equilibrium. They are:

1. Price of factor = value of marginal product. MVP = P_x
2. Marginal product diminishing. MVP ↓
3. Average product diminishing. APP ↓

In the usual terminology a firm is not in equilibrium unless profits are at a maximum. Thus, fulfillment of these conditions is requisite to profit maximization. These conditions are fulfilled when the following conditions hold:[3]

1. $$\frac{dy}{dx} = \frac{P_x}{P_y} \tag{17}$$

2. $$\frac{d^2y}{dx^2} < 0 \tag{18}$$

3. $$\frac{d}{dx}\left(\frac{Y}{X}\right) < 0 \tag{19}$$

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[1] Exceptions are in order for those producers who produce for the sake of enjoyment of participating in the production process and who ignore costs of production and value of production. Exceptions are also in order for those producers who vie with each other for the highest yields per acre, etc.

[2] J. R. Hicks. Value and capital. 2nd edition. Oxford. The Clarendon Press. 1950. p. 82.

[3] In these conditions X and Y denote quantity of factor and quantity of product; $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and $\frac{d}{dx}\left(\frac{Y}{X}\right)$ have the usual mathematical interpretation; and P_x and P_y denote price of factor and price of product.

In terms of the corn-nitrogen data, this implies that profits from the use of nitrogen will be maximized if and only if the rate of nitrogen used is such that these three conditions are met.

Procedure under static conditions

If there were only one response function and prices were certain, a corn producer could lay plans once and for all for the use of nitrogen in the production process. The rate of nitrogen which satisfied conditions (17)-(19) would be the rate which would be applied every year with exactly the same results. If no rate of nitrogen satisfied these conditions, the application of any rate of nitrogen whatsoever would reduce profits from the corn enterprise.[1] However, as was pointed out earlier, the producer is not faced with a single response function, but with a distribution of response functions.[2] Hence, this must be taken into account in planning the application of nitrogen to corn.

Procedure under perfect knowledge

Even though the corn producer is faced with a distribution of nitrogen response functions, he may still determine the rate of nitrogen which would

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[1] The unrealistic cases of constant and increasing marginal productivity throughout the range of possible applications of nitrogen are ignored.

[2] That there is an entire distribution of response functions is well demonstrated by the data in Table D in the Appendix. Here the null hypothesis is tested that the regression coefficients for the years for which linear functions were chosen all come from the same parent population. Since the null hypothesis is rejected at the 1 percent level of probability, it may be concluded with small chance of error that the response functions are actually different. Evidence that the production functions are different is more conclusive since the null hypothesis tested does not consider that the Y-intercept is different for the different production functions.

be most profitable[1] for any given year. However, he must know which response function will be realized for that year and prices of corn and nitrogen in order to do so.[2] The planning procedure is exactly the same as under static conditions, except that the most profitable rate is not constant from year to year. Hence, it must be determined for each year individually.

Preparatory to determining the rates of nitrogen which would have been most profitable under perfect knowledge for the period under study, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, and $\frac{d}{dx}\left(\frac{Y}{X}\right)$ were calculated for each year. The resulting functions, or constants as the case may be, are presented in Table 7.

Corn prices during harvest seasons[3] of the 32-year period averaged \$1.06[4] per bushel. Nitrogen prices[5] averaged \$1.17[6] per $7\frac{1}{2}$ pound increment.

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[1]The most profitable rate of nitrogen application as used here and in all other parts of this study refers to that rate which maximizes profits from the corn enterprise for a given response function. In many farm situations, profits from a farm as a whole may be greater from a smaller rate of nitrogen applied to the corn enterprise. This would be the case when, because of capital limitations, capital had greater marginal productivity invested in resources other than nitrogen.

[2]It is possible to always apply the amount of nitrogen which would maximize profits if the function which would be realized in a given year is known (assuming prices known). Hence, weather control and/or weather prediction together with estimation of production functions are areas in which great dividends might be reaped from additional research.

[3]Use of the price of corn during harvest season avoids the complications arising from differences in costs of storage depending on volume. Additional harvesting costs are ignored.

[4]U. S. Bureau of Agricultural Economics. Agricultural prices. U. S. Department of Agriculture. Compiled from monthly publications.

[5]In the form of Nitrate of Soda.

[6]U. S. Bureau of Agricultural Economics. op. cit. March 1953. p. 37.

Table 7. **First and second derivatives of annual total response functions and first derivatives of the corresponding average response functions, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952**

Year	$\frac{dy}{dx} =$	$\frac{d^2y}{dx^2} =$	$\frac{d}{dx}\left(\frac{Y}{X}\right) =$
1921	$1.478X - .3291X^2$	$1.478 - .6582X$	$.739 - .2194X$
1922	$3.654X - .5844X^2$	$3.654 - 1.1688X$	$1.827 - .3896X$
1923	3.336	0	0
1924	$3.572X - .6912X^2$	$3.572 - 1.3824X$	$1.786 - .4608X$
1925	$2.115X^{-\frac{1}{2}} - .426X$	$-.426 - 1.058X^{-\frac{3}{2}}$	$-.213 - 2.114X^{-\frac{3}{2}}$
1926	$4.182X - .7890X^2$	$4.182 - 1.5780X$	$2.091 - .5260X$
1927	$7.368X - 1.4469X^2$	$7.368 - 2.8938X$	$3.684 - .9646X$
1928	$10.704 - 1.060X$	-1.060	$-.530$
1929	$4.044X - .7302X^2$	$4.044 - 1.4604X$	$2.022 - .4868X$
1930	$3.038X - .5469X^2$	$3.038 - 1.0938X$	$1.519 - .3646X$
1931	$4.814 - .648X$	$-.648$	$-.324$
1932	$4.508X - .7638X^2$	$4.508 - 1.5276X$	$2.254 - .5092X$
1933	$1.308X - .1275X^2$	$1.308 - .2550X$	$.654 - .0850X$
1934	$3.574X - .6759X^2$	$3.574 - 1.3518X$	$1.787 - .4506X$
1935	$4.220X - .6930X^2$	$4.220 - 1.3860X$	$2.110 - .4620X$
1936	$1.914X - .3390X^2$	$1.914 - .6780X$	$.957 - .2260X$
1937	$4.968X - .9330X^2$	$4.968 - 1.8660X$	$2.484 - .6220X$
1938	$3.376X - .4173X^2$	$3.376 - .8346X$	$1.688 - .2782X$
1939	$4.032 - .226X$	$-.226$	$-.113$
1940	$2.874X - .4620X^2$	$2.874 - .9240X$	$1.437 - .3080X$
1941	3.189	0	0
1942	$7.375 - .574X$	$-.574$	$-.287$
1943	$6.004 - .722X$	$-.722$	$-.361$
1944	$2.688X - .5730X^2$	$2.688 - 1.1460X$	$1.344 - .3820X$
1945	$7.811 - .622X$	$-.622$	$-.311$
1946	$5.126X - .8142X^2$	$5.126 - 1.6284X$	$2.563 - .5428X$
1947	$5.482 - .8841X^{\frac{1}{2}}$	$.4420X^{-\frac{3}{2}}$	$.884X^{-\frac{3}{2}}$
1948	$3.866X - .6240X^2$	$3.866 - 1.2480X$	$1.933 - .4160X$
1949	$5.506X - .8658X^2$	$5.506 - 1.7316X$	$2.753 - .5772X$
1950	6.329	0	0
1951	$2.075 + .564X$	$.564$	$.282$
1952	$4.166X - .6390X^2$	$4.166 - 1.2780X$	$2.083 - .4260X$

Use of these average prices as the prices in terms of which plans are laid results in a ratio of 1.10 for the ratio $\frac{P_x}{P_y}$. = 1.10

For the maximum profit conditions (17)-(19) must be fulfilled. But for the type response function $Y = bX$, the following conditions hold:

$$\frac{dy}{dx} > 1.10, \quad \frac{d^2y}{dx^2} = 0, \quad \text{and} \quad \frac{d}{dx}\left(\frac{Y}{X}\right) = 0.$$

If $\frac{dy}{dx} > 1.10$, it is implied that profits will be maximized by an infinitely large application of nitrogen. This is clearly illogical because the function would not have been linear over the possible range of nitrogen application in the beginning. Hence, for these functions it was assumed that the highest rate of nitrogen used in the experiment (six increments of $7\frac{1}{2}$ pounds) was the most profitable rate (Table 8). This assumption was also made for all other cases where the indicated most profitable rate was greater than the highest rate used in the experiment.

For the type functions $Y = bX + cX^2$, $\frac{dy}{dx} = 1.10$ resulted in simple algebraic equations with one unknown. Solution of these equations yielded a value of X at which conditions (18) and (19) were satisfied if the sign of $\underline{c}$ was negative. But, if the sign of $\underline{c}$ were positive neither condition (18) nor (19) was satisfied. In the latter case it was again implied that an infinitely large amount of nitrogen would maximize profits.

For the type functions $Y = bX^2 + cX^3$, $\frac{dy}{dx} = 1.10$ resulted in quadratic equations, the solution of which yielded two values of X. Conditions (18) and (19) were both satisfied at the larger of the two values, but neither at the smaller of the two values.

For the function of the type $Y = bX + cX^{\frac{1}{2}}$, $\frac{dy}{dx} = 1.10$ resulted in a simple algebraic equation with one unknown. Solution of this equation yielded a value of X which did not satisfy conditions (18) and (19).

Table 8. Determination of most profitable rate of nitrogen application for each year, corn at $1.06 per bushel and nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Values[a] of X satisfying the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$	$\frac{d^2y}{dx^2}$ evaluated[b]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[b]
1921	.94 and 3.55	.86 and -.86	.53 and -.04
1922	.32 and 5.94	3.28 and -3.29	1.70 and -.49
1923	None		
1924	.33 and 4.84	3.12 and -3.12	1.63 and -.44
1925	1.49	-1.01	-1.38
1926	.28 and 5.02	3.74 and -3.74	1.94 and -.55
1927	.15 and 4.94	6.93 and -6.93	3.54 and -1.08
1928	9.06	-1.06	-.53
1929	.28 and 5.25	3.64 and -3.62	1.89 and -.53
1930	.39 and 5.16	2.61 and -2.61	1.38 and -.36
1931	5.73	-.65	-.32
1932	.25 and 5.65	4.13 and -4.12	2.13 and -.62
1933	.92 and 9.33	1.07 and -1.07	.58 and -.14
1934	.33 and 4.96	3.13 and -3.13	1.64 and -.45
1935	.27 and 5.82	3.85 and -3.85	1.99 and -.58
1936	.65 and 5.00	1.47 and -1.48	.81 and -.17
1937	.23 and 5.09	4.54 and -4.53	2.34 and -.68
1938	.34 and 7.75	3.09 and -3.09	1.59 and -.47
1939	12.97	-.23	-.11
1940	.41 and 5.81	2.50 and -2.49	1.31 and -.35
1941	None		
1942	10.93	-.57	-.29

[a]Only the values which are real are presented.

[b]Evaluated for the values of X which satisfy the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$ and are real. In the cases where two values satisfying the equation are presented, the second derivative of the response function evaluated at the two values has the same numerical value but has opposite signs. All differences are due to rounding.

Table 8. (Continued)

Year	Values[a] of X satisfying the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$	$\frac{d^2y}{dx^2}$ evaluated[b]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[b]
1943	6.79	-.72	-.36
1944	.45 and 4.24	2.17 and -2.17	1.17 and -.28
1945	10.79	-.62	-.31
1946	.22 and 6.07	4.77 and -4.76	2.44 and -.73
1947	.04	55.25	110.50
1948	.30 and 5.90	3.49 and -3.50	1.81 and -.52
1949	.21 and 6.15	5.14 and -5.14	2.63 and -.80
1950	None		
1951	-1.73	.56	.28
1952	.28 and 6.24	3.81 and -3.81	1.96 and -.58

Again the implication was an infinite amount of nitrogen as the most profitable rate.

For the remaining function, $Y = bX^2 + cX^{\frac{1}{2}}$, $\frac{dy}{dx} = 1.10$ resulted in a cubic equation. From solution of this equation one value of X which was real and which satisfied conditions (18) and (19) was obtained.

Rates of nitrogen which would have maximized profits from the use of nitrogen under conditions of perfect knowledge as to the function which would have been realized are presented in Table 13. From these rates, P_y, P_x, and yields calculated from the response functions, profits from the use of nitrogen each year were calculated. However, in order to make comparisons with profits following various planning procedures under uncertainty regarding the function which would have been realized, a discussion of these details is relegated to a later section.

The corn producer is faced not with perfect knowledge, but with uncertainty as to the responses which will result from given applications of nitrogen. Hence, he must take this uncertainty into consideration in making annual production plans. Each time that responses different from those in terms of which plans were laid are realized, an error has been made. Heady[1] terms these errors or deviations as "expectational errors." He states further that "two types of deviations or errors are important: (1) the error in a single year and (2) the average of errors over time."[2] Which of these two types is weighted more heavily in planning under uncertainty depends upon the psychological make-up of the individual and his capital position.

Among other possibilities for planning under undertainty the decision as to how much nitrogen to apply in a given year may be based on the following:

1. Function realized the past year.
2. Functions realized in some past period (parallel period planning).
3. Best responses realized in some past period.
4. Poorest responses realized in some past period.
5. Mean responses realized in some past period.
6. Median responses realized in some past period.
7. First quartile responses realized in some past period.
8. Third quartile responses realized in some past period.

Many farmers probably use one of the above procedures in laying plans for the year ahead. The list could be extended to cover many other possibilities.

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[1]Earl O. Heady. Economics of agricultural production and resource use. p. 479.

[2]Ibid.

However, these should suffice to obtain some idea of the differences in profits and chances of loss which would result from following different planning procedures.[1]

In order to compare the different planning procedures listed above as to profits and chances of loss, it was necessary to estimate the relationship between mean, median, first quartile, and third quartile responses and the rate of nitrogen applied. Results of fitting the different type functions to the mean, median, first quartile, and third quartile responses are presented in Table 9.[2] In each case the function which gave the best fit was of the type $Y = a + bX^2 + cX^3$. These functions, together with the resulting correlation coefficients, are presented in Table 10 as response to nitrogen functions. Size of the correlation coefficients indicate that the functions selected give an excellent fit to the data.

If plans are laid in terms of the mean, median, first quartile, or third quartile response functions, determination of the rate of nitrogen

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[1]The analysis which follows presupposes knowledge of the functions for the entire period at the beginning of the period, though not the order in which the functions will be realized. Naturally this is at variance with the facts. However, under the assumption that the responses obtained during the period are a random sample from the possible response, the procedure followed should yield useable results.

[2]As pointed out earlier, weighted regression techniques were used in fitting these functions. (See Table E in the Appendix for the estimates of variance and weights used.)

It was realized that estimates of variance used in establishing the weights would not be applicable for the median, first quartile, and third quartile responses. However, in the absence of better methods for estimating weights to be used in fitting functions to these values, the same weights were used as were used in fitting a function to the means.

Table 9. Results of fitting various type functions to the mean, median, first quartile, and third quartile responses[a] for different levels of nitrogen, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Type of function[b]	a	b	c	Weighted reduction due to regression	Weighted mean square deviation from regression	Correlation coefficient[c]
Mean						
1	-1.31	4.356		116,018.98	197.34	.9966
2	-2.33	5.399	-.1799	116,431.12	125.74	.9984
3	1.44	1.816	-.2061	116,698.23	36.70	.9995
4	-4.61	2.881	4.6502	116,257.98	183.45	.9976
5	-8.24	.150	10.9722	116,007.81	266.84	.9966
Median						
1	-4.30	5.512		124,504.00	216.09	.9966
2	-3.38	5.505	-.1712	124,876.91	163.82	.9980
3	.46	1.871	-.2114	125,291.02	25.78	.9997
4	-5.51	3.168	4.2368	124,702.39	221.99	.9973
5	-9.38	.165	11.1719	124,409.59	319.59	.9962
First quartile						
1	-1.75	3.457		73,066.48	184.53	.9950
2	-2.81	4.533	-.1856	73,504.87	99.91	.9980
3	.35	1.491	-.1733	73,715.21	29.79	.9994
4	-5.33	1.860	5.0351	73,346.68	152.64	.9969
5	-7.77	.091	9.2151	73,211.70	197.63	.9960

[a] Response calculated from the functions presented in Table 4 at 1, 2, 3, 4, 5, and 6 increments of nitrogen.

[b] Functions of the type $Y = a + bX$, $Y = a + bX + cX^2$, $Y = a + bX^2 + cX^3$, $Y = a + bX + cX^{1/2}$, and $Y = a + bX^2 + cX^{1/2}$ are denoted by 1, 2, 3, 4, and 5, respectively.

[c] Correlation coefficients greater than .811 and .917 denote significance at the 5 percent and 1 percent probability levels, respectively, for functions of the first type. For the other type functions correlation coefficients greater than .930 and .976 denote significance at the 5 percent and 1 percent levels, respectively.

Table 9. (Continued)

Type of function[b]	a	b	c	Weighted reduction due to regression	Weighted mean square deviation from regression	Correlation coefficient[c]
Third quartile						
1	-1.47	5.225		166,942.76	116.41	.9986
2	-1.51	5.268	-.0074	166,943.46	154.97	.9986
3	2.17	1.943	-.2016	167,248.91	53.16	.9995
4	-.78	5.529	-.9592	166,952.93	151.82	.9986
5	-3.91	.315	10.6899	166,514.74	297.88	.9973

Table 10. Selected planning functions,[a] Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Function	Y =	Correlation coefficient[b]
Best response[c]	$10.704X - .530X^2$	.993
Poorest response[d]	$.739X^2 - .1097X^3$	.997
Mean response	$1.44 + 1.816X^2 - .2061X^3$	.9995
Median response	$.46 + 1.871X^2 - .2114X^3$	.9997
First quartile response	$.35 + 1.491X^2 - .1733X^3$	.9995
Third quartile response	$2.17 + 1.943X^2 - .2016X^3$	.9995

[a] Y denotes response in corn yields in bushels per acre and X denotes nitrogen applied per acre in 7½ pound increments.

[b] Correlation coefficients greater than .949 are significant at the 1 percent level of probability for the first two functions and those greater than .976 are significant at the 1 percent level for the latter four functions.

[c] Highest responses were obtained in the year 1928 throughout the observed range of X.

[d] Lowest responses were obtained in the range of X = 0 to X = 4.2 in the year 1921 and in the range X = 4.2 to X = 6 in the year 1925. Since both functions show decreasing total product in the latter range the function for 1921 is used to represent the lowest responses.

which would be applied each year is made in exactly the same manner as previously presented. The rate which satisfied conditions (17)-(19) for each of the functions is the rate which would be applied each year if planning is on the basis of that function. Determination of these rates is presented in Tables 11 and 12.

Profits or losses from the application of nitrogen were calculated as $P_y(Y) - P_x(X)$. For the condition of perfect knowledge and the various systems of planning outlined earlier, X is presented in Table 13, $P_x(X)$ in Table 14, Y in Table 15, $P_y(Y)$ in Table 16, and profits and losses from the use of nitrogen in Table 17. Finally, profits are arrayed in Table 18 for ease of comparison.

Profits per acre from the use of nitrogen would have averaged \$18.05 under conditions of perfect knowledge. If plans were laid on the basis of the poorest responses realized during the period, profits would have averaged \$11.72 per acre. For the other systems of planning profits would have averaged from \$16.15 to \$16.93 per acre, but losses ranging up to \$3.95 per acre would have been realized in two years of the 32-year period.

Although the system of planning whereby plans are based on the poorest responses resulted in the lowest average profit per acre, it may be preferred to the other planning procedures under certain circumstances. Loss from the use of nitrogen did not occur following this procedure. Hence, in farm situations where capital is limited and chance of loss as well as average gain must be considered, this system of planning might be looked upon with favor.

Table 11. First and second derivatives of selected planning total response functions and first derivatives of the corresponding average response functions, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Type of response function	$\frac{dy}{dx} =$	$\frac{d^2y}{dx^2} =$	$\frac{d}{dx}\left(\frac{Y}{X}\right) =$
Mean	$3.632X - .6183X^2$	$3.632 - 1.2366X$	$1.816 - .4122X$
Median	$3.742X - .6342X^2$	$3.742 - 1.2684X$	$1.871 - .4228X$
First quartile	$2.982X - .5199X^2$	$2.982 - 1.0398X$	$1.491 - .3466X$
Third quartile	$3.886X - .6048X^2$	$3.886 - 1.2096X$	$1.943 - .4032X$

Table 12. Determination of most profitable rates of nitrogen application for selected planning functions, corn at $1.06 per bushel and nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Type of response function	Values of X satisfying the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$	$\frac{d^2y}{dx^2}$ evaluated[a]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[a]
Mean	.32 and 5.55	3.24 and -3.23	1.68 and -.47
Median	.31 and 5.59	3.35 and -3.35	1.74 and -.49
First quartile	.40 and 5.34	2.57 and -2.57	1.35 and -.36
Third quartile	.30 and 6.13	3.52 and -3.53	1.82 and -.53

[a]Evaluated for the values of X which satisfy the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$. The second derivative evaluated at these two values of X are numerically the same but of opposite sign. All differences are due to rounding.

Table 13. Rates of nitrogen which would be used following various planning procedures, corn at $1.06 per bushel and nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure[a]		
	Perfect knowledge	Last year's function[b]	Parallel period[c]
		(7½ pound increments)	
1921	3.55	6.00	5.09
1922	5.94	3.55	6.00
1923	6.00	5.94	6.00
1924	4.84	6.00	5.81
1925	1.49	4.84	6.00
1926	5.02	1.49	6.00
1927	4.94	5.02	6.00
1928	6.00	4.94	4.24
1929	5.25	6.00	6.00
1930	5.16	5.25	6.00
1931	5.73	5.16	6.00
1932	5.65	5.73	5.90
1933	6.00	5.65	6.00
1934	4.96	6.00	6.00
1935	5.82	4.96	6.00
1936	5.00	5.82	6.00
1937	5.09	5.00	3.55
1938	6.00	5.09	5.94
1939	6.00	6.00	6.00
1940	5.81	6.00	4.84
1941	6.00	5.81	1.49
1942	6.00	6.00	5.02

[a] If planning is in terms of the best responses, poorest responses, mean responses, median responses, first quartile responses, or third quartile responses, an annual application of 6.00, 3.55, 5.55, 5.59, 5.34, or 6.00 increments of 7½ pounds would be applied, respectively.

[b] The function realized for 1952 used as the function in terms of which plans are made for 1921.

[c] The second 16 years to parallel with the first 16 and vice versa.

Table 13. (Continued)

Year	Planning procedure[a]		
	Perfect knowledge	Last year's function[b]	Parallel period[c]
		(7½ pound increments)	
1943	6.00	6.00	4.94
1944	4.24	6.00	6.00
1945	6.00	4.24	5.25
1946	6.00	6.00	5.16
1947	6.00	6.00	5.73
1948	5.90	6.00	5.65
1949	6.00	5.90	6.00
1950	6.00	6.00	4.96
1951	6.00	6.00	5.82
1952	6.00	6.00	5.00

Table 14. Cost of nitrogen applied following various planning procedures, corn at $1.06 per bushel and nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure[a] Perfect knowledge	Last year's function	Parallel period
		(dollars)	
1921	4.15	7.02	5.96
1922	6.95	4.15	7.02
1923	7.02	6.95	7.02
1924	5.66	7.02	6.80
1925	1.74	5.66	7.02
1926	5.87	1.74	7.02
1927	5.78	5.87	7.02
1928	7.02	5.78	4.96
1929	6.14	7.02	7.02
1930	6.03	6.14	7.02
1931	6.70	6.04	7.02
1932	6.61	6.70	6.90
1933	7.02	6.61	7.02
1934	5.80	7.02	7.02
1935	6.80	5.80	7.02
1936	5.85	6.81	7.02
1937	5.96	5.85	4.15
1938	7.02	5.96	6.95
1939	7.02	7.02	7.02
1940	6.80	7.02	5.66
1941	7.02	6.80	1.74
1942	7.02	7.02	5.87
1943	7.02	7.02	5.78
1944	4.96	7.02	7.02
1945	7.02	4.96	6.14
1946	7.02	7.02	6.03
1947	7.02	7.02	6.70
1948	6.90	7.02	6.61
1949	7.02	6.90	7.02
1950	7.02	7.02	5.80
1951	7.02	7.02	6.81
1952	7.02	7.02	5.85

[a]If planning were in terms of the best responses, poorest responses, mean responses, median responses, first quartile responses, or third quartile responses, the annual cost of nitrogen would have been $7.02, $4.15, $6.49, $6.54, $6.25, and $7.02, respectively.

Table 15. Annual responses which would have been obtained following various planning procedures, corn at \$1.06 per bushel and nitrogen at \$1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses[a]	Poorest responses	Mean responses	Median responses	First quartile responses
					(bushels)			
1921	4.4	2.9	4.7	2.9	4.4	4.0	4.0	4.4
1922	23.6	14.3	23.7	23.7	14.3	23.0	23.1	22.4
1923	20.0	19.8	20.0	20.0	11.8	18.5	18.6	17.8
1924	15.7	14.6	15.1	14.6	12.2	15.6	15.6	15.8
1925	4.7	4.3	2.6	2.6	5.3	3.4	3.3	3.7
1926	19.4	3.8	18.5	18.5	14.6	19.4	19.4	19.6
1927	31.8	31.8	28.5	28.5	24.9	31.0	30.9	31.6
1928	45.1	39.9	35.9	45.1	31.3	43.1	43.3	42.0
1929	20.5	20.2	20.2	20.2	14.6	20.7	20.7	20.6
1930	15.4	15.5	15.2	15.2	11.0	15.6	15.6	15.6
1931	17.0	16.2	17.3	17.3	13.0	16.7	16.8	16.5
1932	26.1	26.1	26.2	26.1	17.0	25.8	25.9	25.5
1933	14.4	13.2	14.4	14.4	6.3	12.9	13.0	12.2
1934	16.5	15.6	15.6	15.6	12.4	16.5	16.5	16.7
1935	25.9	23.7	26.1	26.1	16.3	25.5	25.6	25.0
1936	9.8	10.1	10.0	10.0	7.0	10.2	10.2	10.1
1937	23.3	23.2	17.4	22.2	17.4	23.3	23.3	23.5
1938	30.8	25.4	30.4	30.8	15.0	28.2	28.4	27.0
1939	20.1	20.1	20.1	20.1	12.9	18.9	19.0	18.3
1940	18.3	18.5	16.2	18.5	11.2	17.9	18.0	17.5
1941	19.1	18.5	4.8	19.1	11.3	17.7	17.8	17.0
1942	33.9	33.9	29.7	33.9	22.6	32.1	32.2	31.2
1943	23.0	23.0	20.8	23.0	16.8	22.2	22.3	21.8
1944	9.6	7.1	7.1	7.1	8.4	8.7	8.6	9.2
1945	35.7	27.5	32.4	35.7	23.8	33.8	33.9	32.8
1946	33.7	33.7	31.0	33.7	20.2	32.5	32.7	31.8
1947	28.6	28.6	27.2	28.6	16.1	26.3	26.5	25.2
1948	24.6	24.7	24.2	24.7	15.1	24.0	24.1	23.4
1949	36.8	36.6	36.8	36.8	21.8	35.5	35.6	34.9
1950	38.0	38.0	31.4	38.0	22.5	35.1	35.4	33.8
1951	22.6	22.6	21.6	22.6	10.9	20.2	20.4	19.1
1952	29.0	29.0	25.5	29.0	16.7	27.7	27.9	27.0

[a]Also third quartile responses.

Table 16. Value of responses which would have been obtained following various planning procedures, corn at $1.06 per bushel, nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses[a]	Poorest responses	Mean responses	Median responses	First quartile responses
	(dollars)							
1921	4.66	3.07	4.98	3.07	4.66	4.24	4.24	4.66
1922	25.02	15.16	25.12	25.12	15.16	24.38	24.49	23.74
1923	21.20	20.99	21.20	21.20	12.51	19.61	19.72	18.87
1924	16.64	15.48	16.01	15.48	12.93	16.54	16.54	16.75
1925	4.98	4.56	2.76	2.76	5.62	3.60	3.50	3.92
1926	20.56	4.03	19.61	19.61	15.48	20.56	20.56	20.78
1927	33.71	33.71	30.21	30.21	26.39	32.86	32.75	33.50
1928	47.81	42.29	38.05	47.81	33.18	45.69	45.90	44.52
1929	21.73	21.41	21.41	21.41	15.48	21.94	21.94	21.84
1930	16.32	16.43	16.11	16.11	11.66	16.54	16.54	16.54
1931	18.02	17.17	18.34	18.34	13.78	17.70	17.81	17.49
1932	27.67	27.67	27.77	27.67	18.02	27.35	27.45	27.03
1933	15.26	13.99	15.26	15.26	6.68	13.67	13.78	12.93
1934	17.49	16.54	16.54	16.54	13.14	17.49	17.49	17.70
1935	27.45	25.12	27.67	27.67	17.28	27.03	27.14	26.50
1936	10.39	10.71	10.60	10.60	7.42	10.81	10.81	10.71
1937	24.70	24.59	18.44	23.53	18.44	24.70	24.70	24.91
1938	32.65	26.92	32.22	32.65	15.90	29.89	30.10	28.62
1939	21.31	21.31	21.31	21.31	13.67	20.03	20.14	19.40
1940	19.40	19.61	17.17	19.61	11.87	18.97	19.08	18.55
1941	20.25	19.61	5.09	20.25	11.98	18.76	18.87	18.02
1942	35.93	35.93	31.48	35.93	23.96	34.03	34.13	33.07
1943	24.38	24.38	22.05	24.38	17.81	23.53	23.64	23.11
1944	10.18	7.53	7.53	7.53	8.90	9.22	9.12	9.75
1945	37.84	29.15	34.34	37.84	25.23	35.83	35.93	34.77
1946	35.72	35.72	32.86	35.72	21.41	34.45	34.66	33.71
1947	30.32	20.32	28.83	30.32	17.07	27.88	28.09	26.71
1948	26.08	26.18	25.65	26.18	16.01	25.44	25.55	24.80
1949	39.01	38.80	39.01	39.01	23.11	37.63	37.74	36.99
1950	40.28	40.28	33.28	40.28	23.85	37.21	37.52	35.83
1951	23.96	23.96	22.90	23.96	11.55	21.41	21.62	20.25
1952	30.74	30.74	27.03	30.74	17.70	29.36	29.57	28.62

[a]Also third quartile responses.

Table 17. Profits from use of nitrogen following various planning procedures, corn at $1.06 per bushel and nitrogen at $1.17 per $7\frac{1}{2}$ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses[a]	Poorest responses	Mean responses	Median responses	First quartile responses
					(dollars)			
1921	.51	-3.95	-.98	-3.95	.51	-2.25	-2.30	-1.59
1922	18.07	11.01	18.10	18.10	11.01	17.89	17.95	17.49
1923	14.18	14.04	14.18	14.18	8.36	13.12	13.18	12.62
1924	10.98	8.46	9.21	8.46	8.78	10.05	10.00	10.50
1925	3.24	-1.10	-4.26	-4.26	1.47	-2.89	-3.04	-2.33
1926	14.69	2.29	12.59	12.59	11.33	14.07	14.02	14.53
1927	27.93	27.84	23.19	23.19	22.24	26.37	26.21	27.25
1928	40.79	36.51	33.09	40.79	29.03	39.20	39.36	38.27
1929	15.59	14.39	14.39	14.39	11.33	15.45	15.40	15.59
1930	10.29	10.29	9.09	9.09	7.51	10.05	10.00	10.29
1931	11.32	11.13	11.32	11.32	9.63	11.21	11.27	11.24
1932	21.06	20.97	20.87	20.65	13.87	20.86	20.91	20.78
1933	8.24	7.38	8.24	8.24	2.53	7.18	7.24	6.68
1934	11.69	9.52	9.52	9.52	8.99	11.00	10.95	11.45
1935	20.65	19.32	20.65	20.65	13.13	20.54	20.60	20.25
1936	4.54	3.90	3.58	3.58	3.27	4.32	4.27	4.46
1937	18.74	18.74	14.29	16.51	14.29	18.21	18.16	18.66
1938	25.63	20.96	25.27	25.63	11.75	23.40	23.56	22.37
1939	14.29	14.29	14.29	14.29	9.52	13.54	13.60	13.15
1940	12.60	12.59	11.51	12.59	7.72	12.48	12.54	12.30
1941	13.23	12.81	3.35	13.23	7.83	12.27	12.33	11.77
1942	28.91	28.91	25.61	28.91	19.81	27.54	27.59	26.82
1943	17.36	17.26	16.27	17.36	13.66	17.04	17.10	16.86
1944	5.22	.51	.51	.51	4.75	2.73	2.58	3.50
1945	30.82	24.19	28.20	30.82	21.08	29.34	29.39	28.52
1946	28.70	28.70	26.83	28.70	17.26	27.96	28.12	27.46
1947	23.30	23.30	22.13	23.30	12.92	21.39	21.55	20.46
1948	19.18	19.16	19.04	19.16	11.86	18.95	19.01	18.55
1949	31.99	31.90	31.99	31.99	18.96	31.14	31.20	30.74
1950	33.26	33.26	27.48	33.26	19.70	30.72	30.98	29.58
1951	16.94	16.94	16.09	16.94	7.40	14.92	15.08	14.00
1952	23.72	23.72	21.18	23.72	13.55	22.87	23.03	22.37

[a]Also third quartile responses.

Table 18. Array of profits from use of nitrogen following various planning procedures, corn at $1.06 per bushel and nitrogen at $1.17 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Perfect knowledge	Last year's function	Parallel period	Best responses[a]	Poorest responses	Mean responses	Median responses	First quartile responses
			(dollars)				
.51	-3.95	-4.26	-4.26	.51	-2.89	-3.04	-2.33
3.24	-1.10	-.98	-3.95	1.47	-2.25	-2.30	-1.59
4.54	.51	.51	.51	2.53	2.73	2.58	3.50
5.22	2.29	3.35	3.58	3.27	4.32	4.27	4.46
8.24	3.90	3.58	8.24	4.75	7.18	7.24	6.68
10.29	7.38	8.24	8.46	7.40	10.05	10.00	10.29
10.98	8.46	9.09	9.09	7.51	10.05	10.00	10.50
11.32	9.52	9.21	9.52	7.72	11.00	10.95	11.24
11.69	10.29	9.52	11.32	7.83	11.21	11.27	11.45
12.60	11.01	11.32	12.59	8.36	12.27	12.33	11.77
13.23	11.13	11.51	12.59	8.78	12.48	12.54	12.38
14.18	12.59	12.59	13.23	8.99	13.12	13.18	12.62
14.29	12.81	14.18	14.18	9.52	13.54	13.60	13.15
14.69	14.04	14.29	14.29	9.63	14.07	14.02	14.00
15.59	14.29	14.29	14.39	11.01	14.92	15.08	14.53
16.94	14.39	14.39	16.51	11.33	15.45	15.40	15.59
17.36	16.94	16.09	16.94	11.33	17.04	17.10	16.86
18.07	17.36	16.27	17.36	11.75	17.89	17.95	17.49
18.74	18.74	18.10	18.10	11.86	18.21	18.16	18.55
19.18	19.16	19.04	19.16	12.92	18.95	19.01	18.66
20.65	19.32	20.65	20.65	13.13	20.54	20.60	20.25
21.06	20.96	20.87	20.65	13.55	20.86	20.91	20.46
23.30	20.97	21.18	23.19	13.66	21.39	21.55	20.78
23.72	23.30	22.13	23.30	13.87	22.87	23.03	22.37
25.63	23.72	23.19	23.72	14.29	23.40	23.56	22.37
27.93	24.19	25.27	25.63	17.26	26.37	26.21	26.82
28.70	27.84	25.61	28.70	18.96	27.54	27.59	27.25
28.91	28.70	26.83	28.91	19.70	27.96	28.12	27.46
30.82	28.91	27.48	30.82	19.81	29.34	29.39	28.52
31.99	31.90	28.20	31.99	21.08	30.72	30.98	29.58
33.26	33.26	31.99	33.26	22.24	31.14	31.20	30.74
40.79	36.51	33.09	40.79	29.03	39.20	39.36	38.27
			Mean				
18.05	16.23	16.15	16.98	11.72	16.90	16.93	16.71

[a]Also third quartile.

Results which would have been obtained under perfect knowledge and from following the various planning procedures probably would have been less alike if the experiment had involved higher rates of nitrogen. When a rate of nitrogen greater than six increments was indicated as the most profitable rate, only six increments was used in further calculations. In addition, regardless of the system of planning, except for planning in terms of the poorest responses, the most profitable rates of nitrogen did not in general differ much from six increments. Therefore, it is to be expected that results obtained would be very similar.

The most profitable rate of nitrogen application depends upon the price ratio $\frac{P_x}{P_y}$ as well as productivity of nitrogen. Hence, by manipulating this ratio the calculated most profitable rate of application can be made to fall within the range of observations as long as decreasing marginal productivity is encountered in this range. This procedure was used to make the calculated most profitable rates of application more dependent upon the planning procedure.

Effect of changing price ratio

A price ratio of 3.30[1] was found to change the rates of nitrogen which would maximize profits for the individual years and for the planning functions enough so that most of them fell within the range of observations. Determination of the most profitable rate of application under this price ratio for the individual years and for the planning functions (Tables 19 and 20) was made in exactly the same fashion as outlined above for the other ratio.

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[1] Price of corn the same as before, \$1.06 per bushel. Price of nitrogen \$3.50 per 7½ pound increment.

Table 19. Determination of most profitable rate of nitrogen application for each year, corn at \$1.06 per bushel and nitrogen at \$3.50 per $7\frac{1}{2}$ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Values[a] of X satisfying the equation $\frac{dy}{dx} = \frac{Px}{Py}$	$\frac{d^2y}{dx^2}$ evaluated[b]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[b]
1921	None		
1922	1.10 and 5.16	2.37 and -2.38	1.40 and -.18
1923	None		
1924	1.20 and 3.96	1.91 and -1.90	1.23 and -.04
1925	.37	-5.13	-9.61
1926	.96 and 4.24	2.67 and -2.67	1.59 and -.19
1927	.50 and 4.60	5.92 and -5.94	3.20 and -.75
1928	6.99	-1.06	-.53
1929	.99 and 4.54	2.60 and -2.59	1.54 and -.19
1930	1.48 and 4.07	1.42 and -1.41	.98 and .04
1931	2.34	-.65	-.32
1932	.86 and 5.05	3.19 and -3.21	1.82 and -.32
1933	4.47 and 5.78	.17 and -.17	.27 and .16
1934	1.19 and 4.10	1.97 and -1.97	1.25 and -.06
1935	.92 and 5.17	2.94 and -2.95	1.68 and -.28
1936	None		
1937	.78 and 4.55	3.51 and -3.52	2.00 and -.35
1938	1.14 and 6.95	2.42 and -2.42	1.37 and -.25
1939	3.24	-.23	-.11
1940	1.52 and 4.70	1.47 and -1.47	.97 and -.01
1941	None		
1942	7.10	-.57	-.29

[a] Only the values which are real are presented.

[b] Evaluated for the values of X which satisfy the equation $\frac{dy}{dx} = \frac{Px}{Py}$ and are real. In the cases where two values satisfying the equation are presented, the second deviation of the response function evaluated at the two values has the same numerical value but has opposite signs. All differences are due to rounding.

Table 19. (Continued)

Year	Values[a] of X satisfying the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$	$\frac{d^2y}{dx^2}$ evaluated[b]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[b]
1943	3.74	-.72	-.36
1944	None		
1945	7.25	-.62	-.31
1946	.73 and 5.57	3.94 and -3.94	2.17 and -.46
1947	.16	6.91	13.81
1948	1.02 and 5.17	2.59 and -2.59	1.51 and -.22
1949	.67 and 5.69	4.35 and -4.35	2.37 and -.53
1950	None		
1951	2.17	.56	.28
1952	.92 and 5.60	2.99 and -2.99	1.69 and -.30

Table 20. Determination of most profitable rate of nitrogen application for selected planning functions, corn at \$1.06 per bushel and nitrogen at \$3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Type of response function	Values of X satisfying the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$	$\frac{d^2y}{dx^2}$ evaluated[a]	$\frac{d}{dx}\left(\frac{Y}{X}\right)$ evaluated[a]
Mean	1.12 and 4.75	2.25 and -2.25	1.35 and -.14
Median	1.08 and 4.82	2.37 and -2.37	1.41 and -.17
First quartile	1.50 and 4.24	1.42 and -1.43	.97 and -.98
Third quartile	1.01 and 5.42	2.66 and -2.67	1.54 and -.24

[a]Evaluated for the values of X which satisfy the equation $\frac{dy}{dx} = \frac{P_x}{P_y}$. The second derivative evaluated at these two values of X are numerically the same but of opposite sign. All differences are due to rounding.

Three situations developed using this price ratio, however, that were not encountered earlier. First, the price ratio was greater than $\frac{dy}{dx}$ for one of the linear functions (1941). Second, solution of the quadratic equation $\frac{dy}{dx} = 3.30$ resulted in two roots which were imaginary for three functions of the type $Y = bX^2 + cX^3$ (1921, 1936 and 1944). Third, conditions (17) and (18) were fulfilled but condition (19) was not for two functions of the type $Y = bX^2 + cX^3$ (1930 and 1933). The first and second situations indicate that regardless of the rate, application of nitrogen would result in losses from its application. Consequently, the most profitable rate is zero. The third situation indicates that although marginal productivity of nitrogen at the indicated rates is such that value of the marginal product equals price of nitrogen, average productivity is below that required for the application of nitrogen to be profitable at all. Other results of changing the price ratio are presented in Tables 21-26.

If planning were done on the basis of perfect knowledge, losses from the application of nitrogen would never occur. However, using the price ratio 3.30 there would have been six years during the period in which no nitrogen would have been applied. Profits from the use of nitrogen would have ranged from zero to $26.81, and would have averaged $6.78 per acre per year. Since the producer of corn must plan in the face of uncertainty, however, these figures are useful only as a standard with which results of the various planning procedures may be compared.

Planning on the basis of the function actually realized the preceding year would have resulted in losses ranging up to $15.47 in ten years of the period under study and in profits ranging up to $24.18 in sixteen years. As an average for the entire period this system of planning would have resulted in a profit of $2.72 per acre.

Table 21. Rates of nitrogen which would be used following various planning procedures,[a] corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure[a]		
	Perfect knowledge	Last year's function[b]	Parallel period[c]
	(7½ pound increments)		
1921	.00	5.60	4.55
1922	5.16	.00	6.00
1923	6.00	5.16	3.24
1924	3.96	6.00	4.70
1925	.37	3.96	.00
1926	4.34	.37	6.00
1927	4.60	4.34	3.74
1928	6.00	4.60	.00
1929	4.54	6.00	6.00
1930	.00	4.54	5.57
1931	2.34	.00	6.00
1932	5.05	2.34	5.17
1933	.00	5.05	5.69
1934	4.10	.00	6.00
1935	5.17	4.10	6.00
1936	.00	5.17	5.60
1937	4.55	.00	.00
1938	6.00	4.55	5.16
1939	3.24	6.00	6.00
1940	4.70	3.24	3.96
1941	.00	4.70	.37
1942	6.00	.00	4.34

[a]If planning is in terms of the best responses, poorest responses, mean responses, median responses, first quartile responses, or third quartile responses, an annual application of 6.00, .00, 4.75, 4.82, 4.24, and 5.42 increments of 7½ pounds would be applied, respectively.

[b]The function realized for 1952 used as the function in terms of which plans are made for 1921.

[c]The second 16 years to parallel with the first 16 and vice versa.

Table 21. (Continued)

Year	Planning procedure[a]		
	Perfect knowledge	Last year's function[b]	Parallel period[c]
		($7\frac{1}{2}$ pound increments)	
1943	3.74	6.00	4.60
1944	.00	3.74	6.00
1945	6.00	.00	4.54
1946	5.57	6.00	.00
1947	6.00	5.57	2.34
1948	5.17	6.00	5.05
1949	5.69	5.17	.00
1950	6.00	5.69	4.10
1951	6.00	6.00	5.17
1952	5.60	6.00	.00

Table 22. Cost of nitrogen applied following various planning procedures, corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure[a] Perfect knowledge	Last year's function	Parallel period
		(dollars)	
1921	.00	19.60	15.93
1922	18.06	.00	21.00
1923	21.00	18.06	11.34
1924	13.86	21.00	16.45
1925	1.30	13.86	.00
1926	15.19	1.30	21.00
1927	16.10	15.19	13.09
1928	21.00	16.10	.00
1929	15.89	21.00	21.00
1930	.00	15.89	19.50
1931	8.19	.00	21.00
1932	17.68	8.19	18.10
1933	.00	17.68	19.93
1934	14.35	.00	21.00
1935	18.10	14.35	21.00
1936	.00	18.10	19.60
1937	15.93	.00	.00
1938	21.00	15.93	18.06
1939	11.34	21.00	21.00
1940	16.45	11.34	13.86
1941	.00	16.45	1.30
1942	21.00	.00	15.19
1943	13.09	21.00	16.10
1944	.00	13.09	21.00
1945	21.00	.00	15.89
1946	19.50	21.00	.00
1947	21.00	19.50	8.19
1948	18.10	21.00	17.68
1949	19.93	18.10	.00
1950	21.00	19.93	14.35
1951	21.00	21.00	18.10
1952	19.60	21.00	.00

[a]If planning were in terms of the best responses, poorest responses, mean responses, median responses, first quartile responses, or third quartile responses, the annual cost of nitrogen would have been $21.00, $.00, $16.63, $16.87, $14.84, and $18.97, respectively.

Table 23. Annual responses which would have been obtained following various planning procedures, corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses	Mean responses	Median responses	First quartile responses	Third quartile responses
	(bushels)							
1921	.0	3.9	5.0	2.9	4.9	4.9	4.9	4.2
1922	21.9	.0	23.7	23.7	20.3	20.6	18.0	22.7
1923	20.0	17.2	10.8	20.0	15.8	16.1	14.1	18.0
1924	13.7	14.6	15.5	14.6	15.6	15.7	14.5	15.8
1925	2.5	5.1	.0	2.6	4.4	4.3	4.9	3.6
1926	17.9	.3	18.5	18.5	19.0	19.1	17.5	19.6
1927	31.0	30.0	26.3	28.5	31.4	31.6	29.5	31.4
1928	45.1	38.0	.0	45.1	38.9	39.3	35.9	42.4
1929	18.9	20.2	20.2	20.2	19.5	19.7	17.8	20.6
1930	.0	14.2	15.6	15.2	14.7	14.9	13.4	15.6
1931	9.5	.0	17.3	17.3	15.6	15.7	14.6	16.6
1932	24.7	9.1	25.0	26.1	23.6	23.9	21.1	25.7
1933	.0	11.2	13.3	14.4	10.2	10.4	8.5	12.4
1934	14.5	.0	15.6	15.6	16.2	16.3	14.9	16.6
1935	24.5	19.5	26.1	26.1	22.9	23.2	20.3	25.2
1936	.0	10.0	10.2	10.0	9.5	9.6	8.6	10.1
1937	22.1	.0	.0	22.2	22.7	22.9	21.0	23.5
1938	30.8	21.8	25.8	30.8	23.2	23.6	19.7	27.4
1939	11.9	20.1	20.1	20.1	16.6	16.8	15.1	18.5
1940	15.8	9.8	13.0	18.5	15.9	16.1	14.1	17.7
1941	.0	15.0	1.2	19.1	15.1	15.4	13.5	17.3
1942	33.9	.0	26.6	33.9	28.6	28.9	26.1	31.5
1943	17.4	23.0	20.0	23.0	20.4	20.6	19.0	21.9
1944	.0	8.8	7.1	7.1	9.9	9.8	9.6	9.1
1945	35.7	.0	29.1	35.7	30.1	30.4	27.5	33.2
1946	32.6	33.7	.0	33.7	28.7	29.2	25.4	32.1
1947	28.6	26.4	10.1	28.6	22.2	22.5	19.6	25.6
1948	22.9	24.7	22.5	24.7	21.3	21.6	18.9	23.7
1949	36.0	33.7	.0	36.8	31.2	31.6	27.5	34.9
1950	38.0	36.0	25.9	38.0	30.1	30.5	26.8	34.3
1951	22.6	22.6	18.3	22.6	16.2	16.6	13.9	19.5
1952	27.9	29.0	.0	29.0	24.2	24.5	21.2	27.3

Table 24. Value of responses which would have been obtained following various planning procedures, corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses	Mean responses	Median responses	First quartile responses	Third quartile responses
	(dollars)							
1921	.00	4.13	5.30	3.07	5.19	5.19	5.19	4.45
1922	23.21	.00	25.12	25.12	21.52	21.84	19.08	24.06
1923	21.20	18.23	11.45	21.20	16.75	17.07	14.95	19.08
1924	14.52	15.48	16.43	15.48	16.54	16.64	15.37	16.75
1925	2.65	5.41	.00	2.76	4.66	4.56	5.19	3.82
1926	18.97	.32	19.61	19.61	20.14	20.25	18.55	20.78
1927	32.86	31.80	27.88	30.21	33.28	33.50	31.27	33.28
1928	47.81	40.28	.00	47.81	41.23	41.66	38.05	44.94
1929	20.03	21.41	21.41	21.41	20.67	20.88	18.87	21.84
1930	.00	15.05	16.54	16.11	15.58	15.79	14.20	16.54
1931	10.07	.00	18.34	18.34	16.54	16.64	15.48	17.60
1932	26.18	9.65	26.50	27.67	25.02	25.33	22.37	27.24
1933	.00	11.87	14.10	15.26	10.81	11.02	9.01	13.14
1934	15.37	.00	16.54	16.54	17.17	17.28	15.79	17.60
1935	25.97	20.67	27.67	27.67	24.27	24.59	21.52	26.71
1936	.00	10.60	10.81	10.60	10.07	10.18	9.12	10.71
1937	23.43	.00	.00	23.53	24.06	24.27	22.26	24.91
1938	32.65	23.11	27.35	32.65	24.59	25.02	20.88	29.01
1939	12.61	21.31	21.31	21.31	17.60	17.81	16.01	19.61
1940	16.65	10.39	13.78	19.61	16.85	17.07	14.95	18.76
1941	.00	15.90	1.27	20.25	16.01	16.32	14.31	18.34
1942	35.93	.00	28.20	35.93	30.32	30.63	27.67	33.39
1943	18.44	24.38	21.20	24.38	21.62	21.84	20.14	23.21
1944	.00	9.33	7.53	7.53	10.49	10.39	10.18	9.65
1945	37.84	.00	30.85	37.84	31.91	32.22	29.15	35.19
1946	34.56	35.72	.00	35.72	20.42	30.95	26.92	34.03
1947	30.32	27.98	10.71	30.32	25.53	23.85	20.78	27.14
1948	24.27	26.18	23.85	26.18	22.58	22.90	20.03	25.12
1949	38.16	35.72	.00	39.01	33.07	33.50	29.15	36.99
1950	40.28	38.16	27.45	40.28	31.91	32.33	28.41	36.36
1951	23.96	23.96	19.40	23.96	17.17	17.60	14.73	20.67
1952	29.57	30.74	.00	30.74	25.65	25.97	22.47	28.94

Table 25. Profits from use of nitrogen following various planning procedures, corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Planning procedure							
	Perfect knowledge	Last year's function	Parallel period	Best responses	Mean responses	Median responses	First quartile responses	Third quartile responses
					(dollars)			
1921	.00	-15.47	-10.63	-17.93	-11.44	-11.68	-9.65	-14.52
1922	5.15	.00	4.12	4.12	4.89	4.97	4.24	5.09
1923	.20	.17	.11	.20	.12	.20	.11	.11
1924	.66	-5.52	-.02	-5.52	-.09	-.23	.53	-2.22
1925	1.35	-8.45	.00	-18.24	-11.97	-12.31	-9.65	-15.15
1926	3.78	-.98	-1.39	-1.39	3.51	3.38	3.71	1.81
1927	16.76	16.61	14.79	9.21	16.65	16.63	16.43	14.31
1928	26.81	24.18	.00	26.81	24.60	24.79	23.21	25.97
1929	4.14	.41	.41	.41	4.04	4.01	4.03	2.87
1930	.00	-.84	-2.96	-4.89	-1.05	-1.08	-.64	-2.43
1931	1.88	.00	-2.66	-2.66	-.09	-.23	.64	-1.37
1932	8.50	1.46	8.40	6.67	8.39	8.46	7.53	8.27
1933	.00	-5.81	-5.83	-5.74	-5.82	-5.85	-5.83	-5.83
1934	1.02	.00	-4.46	-4.46	.54	.41	.95	-1.37
1935	7.87	6.32	6.67	6.67	7.64	7.72	6.68	7.74
1936	.00	-7.50	-8.79	-10.40	-6.56	-6.69	-5.72	-8.26
1937	7.50	.00	.00	2.53	7.43	7.40	7.42	5.94
1938	11.65	7.18	9.29	11.65	7.96	8.15	6.04	10.07
1939	1.27	.31	.31	.31	.97	.94	1.17	.64
1940	.30	-.95	-.08	-1.39	.22	.20	.11	-.21
1941	.00	-.55	-.03	-.75	-.62	-.55	-.53	-.63
1942	14.93	.00	13.01	14.93	13.69	13.76	12.83	14.42
1943	5.35	3.38	5.10	3.38	4.99	4.97	5.30	4.24
1944	.00	-3.76	-13.47	-13.47	-6.14	-6.48	-4.66	-9.32
1945	16.84	.00	14.96	16.84	15.28	15.35	14.31	16.22
1946	15.06	14.72	.00	14.72	13.79	14.08	12.08	15.06
1947	9.32	8.48	2.52	9.32	6.90	6.98	5.94	8.17
1948	6.17	5.18	6.17	5.18	5.95	6.03	5.19	6.15
1949	18.23	17.62	.00	18.01	16.44	16.63	14.31	18.02
1950	19.28	18.23	13.10	19.28	15.28	15.46	13.57	17.39
1951	2.96	2.96	1.30	2.96	.54	.76	-.11	1.70
1952	9.97	9.74	.00	9.74	9.02	9.10	7.63	9.97

Table 26. Array of profits from use of nitrogen following various planning procedures, corn at $1.06 per bushel and nitrogen at $3.50 per 7½ pound increment, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Perfect knowledge	Last year's function	Parallel period	Best responses	Mean responses	Median responses	First quartile responses	Third quartile responses
			(dollars)				
.00	-15.47	-13.47	-18.24	-11.97	-12.31	-9.65	-15.15
.00	-8.45	-10.63	-17.93	-11.44	-11.68	-9.65	-14.52
.00	-7.50	-8.79	-13.47	-6.56	-6.69	-5.83	-9.32
.00	-5.81	-5.83	-10.40	-6.14	-6.48	-5.72	-8.26
.00	-5.52	-4.46	-5.74	-5.82	-5.85	-4.66	-5.83
.00	-3.76	-2.96	-5.52	-1.05	-1.08	-.64	-2.43
.20	-.98	-2.66	-4.89	-.62	-.55	-.53	-2.22
.30	-.95	-1.39	-4.46	-.09	-.23	-.11	-1.37
.66	-.84	-.08	-2.66	-.09	-.23	.11	-1.37
1.02	-.55	-.03	-1.39	.12	.20	.11	-.63
1.27	.00	-.02	-1.39	.22	.20	.53	-.21
1.35	.00	.00	-.75	.54	.41	.64	.11
1.88	.00	.00	.20	.54	.73	.95	.64
2.96	.00	.00	.31	.97	.94	1.17	1.70
3.78	.00	.00	.41	3.51	3.38	3.71	1.81
4.14	.00	.00	2.53	4.04	4.01	4.03	2.87
5.15	.17	.00	2.96	4.89	4.97	4.24	4.24
5.35	.31	.11	3.38	4.99	4.97	5.19	5.09
6.17	.41	.31	4.12	5.95	6.03	4.30	5.94
7.50	1.46	.41	5.18	6.90	6.98	5.94	6.15
7.87	2.96	1.30	6.67	7.43	7.40	6.04	7.74
8.50	3.38	2.52	6.67	7.64	7.72	6.68	8.17
9.32	5.18	4.12	9.21	7.96	8.15	7.42	8.27
9.97	6.32	5.10	9.32	8.39	8.46	7.53	9.97
11.65	7.18	6.17	9.74	9.02	9.10	7.63	10.07
14.93	8.48	6.67	11.65	13.69	13.76	12.08	14.31
15.06	9.74	8.40	14.72	13.79	14.08	12.83	14.42
16.76	14.72	9.29	14.93	15.28	15.35	13.57	15.06
16.84	16.61	13.01	16.84	15.28	15.46	14.31	16.22
18.23	17.62	13.10	18.01	16.44	16.63	14.31	17.39
19.28	18.23	14.79	19.28	16.65	16.63	16.43	18.02
26.81	24.18	14.96	26.81	24.60	24.79	23.21	25.97
			Mean				
6.78	2.72	1.56	3.00	4.53	4.54	4.29	4.15

If planning were done on the basis of the functions realized the second sixteen years being the same functions and in the same order as those realized the first sixteen years, and vice versa, losses would have occurred in eleven years and profits would have been realized in fifteen years. The greatest loss would have been $13.47 and the highest profit $14.96. Average profit for the period would have been $1.56 per acre.

Planning on the basis of the best responses realized during the period would have resulted in the widest range in profits of any of the systems of planning (a range of -$18.24 to $26.81). Also, this system would have resulted in the largest number of years in which losses would have occurred (twelve years). However, the average profit per acre ($3.00) would have been slightly higher than if plans were laid on the basis of last year's function and considerably higher than if the parallel period procedure were used.

Under the price ratio 3.30 no application of nitrogen would have been made if planning were based on the poorest responses obtained during the period. Consequently, there would not have been neither losses nor profits from the application of nitrogen.

Application of nitrogen on the basis of the mean response function and on the basis of the median response function would have yielded similar results. Profits would have averaged $4.53 per acre for the first procedure and $4.54 for the second. In addition, both planning procedures would have resulted in losses in nine years. The largest loss would have been $11.97 for the mean system and $12.31 for the median system. The greatest profit would have been $24.60 and $24.79 for the mean and median systems, respectively.

If planning were on the basis of first quartile responses, losses ranging up to $9.65 would have occurred in eight years. Profits would have ranged up to $23.21 and would have averaged $4.29 per acre. Profits would have averaged $4.15 per acre had plans been based on the third quartile response function. The largest profit under this system was $25.97. Losses would have occurred in eleven years with $15.15 per acre as the greatest loss.

The above comparison indicates that if a corn producer wished to maximize profits from the application of nitrogen, he should plan in terms of the mean or median responses. Being just a bit less optimistic than this would appear to decrease the chance for loss in greater proportion than average profits.

It appears that a fairly wide range of nitrogen application would result in about the same average profit. (Compare average profits from 4.24 increments of nitrogen, first quartile; 4.82 increments, median; and 5.42 increments, third quartile.) Should the corn producer have limited capital with other alternative uses, it seems that planning in terms of first quartile responses should yield satisfactory results. An annual investment of $14.84 in nitrogen would have resulted in an average profit of $4.29 per acre or a return of 29 percent. Percentage returns were 27, 27, 22, 20, 14, and 11 for the mean, median, third quartile, last year's function, best responses, and parallel period systems, respectively. In addition, chance of loss and size of loss when losses were realized were less for the first quartile system.

These comparisons of and conclusions reached regarding the various planning procedures are no doubt affected by the unrealistic price ratio used in the computations. Therefore, these comparisons and conclusions cannot be accepted at face value. It was pointed out earlier that this study is not a study upon which recommendations to farmers will be based, but a study of an exploratory nature to point out uses which could be made of an estimation of a sample of production functions. Interpretation of all results obtained should be made in this light.

SUMMARY AND CONCLUSIONS

Production plans for agricultural commodities must be made at one point in time for production at some future date. Outcome of agricultural production processes depends upon weather conditions and weather conditions cannot be controlled to any appreciable extent or even predicted for any considerable period ahead. Thus, agricultural producers are faced with uncertainty at the time production plans are made as to the outcome of production processes.

In order to make intelligent production plans in the face of uncertainty, the producer must have some idea of possible outcomes from different production practices and their frequency of occurrence. He must, in most instances, consider chance for loss as well as average profit. In this study several procedures which may be used in planning under uncertainty were compared as to chance of loss and average profit from the application of nitrogen to corn. Because of limitations of the data, however, this study is considered to be of an exploratory nature. It is not a study upon which recommendations to farmers will be based.

Data used for the study were obtained from Agronomy Department files and publications and consisted of per acre yields of corn on experimental plots at the Delta Branch Experiment Station, Stoneville, Mississippi for the period 1921-1952. Rates of nitrogen applied were 0-45 pounds per acre in $7\frac{1}{2}$ pound increments. From these data production functions of the form $Y = f(X_1 | X_2 | X_3, X_4 \ldots X_n)$ where Y denotes corn yields, X_1 denotes nitrogen applied, X_2 denotes weather conditions, and X_3, $X_4 \ldots X_n$ denote the other factors of production were estimated. (The vertical bars indicate that all factors of production other than nitrogen applied were held constant within years and all factors except nitrogen applied and weather conditions were held constant between years.) These functions were then used to estimate a sample of the possible outcomes of applying levels of nitrogen ranging from 0-45 pounds.

In the actual fitting of the functions each of the type functions $Y = a + bX$, $Y = a + bX + cX^2$, $Y = a + bX^2 + cX^3$, $Y = a + bX + cX^{\frac{1}{2}}$, and $Y = a + bX^2 + cX^{\frac{1}{2}}$ were fitted to the data for each year. Selection of the best fit for the individual years was made on the basis of the smallest mean square of the deviations from regression and logic. This resulted in three, seven, and twenty functions, respectively, of the first three types and one each of the other two. These nitrogen production functions were then transformed into nitrogen response functions by subtracting the constant <u>a</u> in each production function from that function. Correlation coefficients for the selected functions were, with one exception, all significant at the 1 percent level. The remaining one was significant at the 5 percent level.

For purposes of comparing several systems of planning functions of the above types were fitted to the mean, median, first quartile, and third quartile of the responses at the different levels of nitrogen. Selection of the type function giving the best fit on the basis of the smallest mean square of the deviations from regression resulted in the type function $Y = a + bX^2 + cX^3$ being selected in each case. Correlation coefficients were significant at the 1 percent level of probability.

Production is in most instances carried on for sake of profit. For profits to be maximized the following three conditions must be fulfilled: (1) price of factor = value of marginal product, (2) marginal product diminishing, and (3) average product diminishing. Under static conditions the producer of corn would select that rate of nitrogen which would satisfy these three conditions, if such a rate should exist, and apply that rate every year with exactly the same results. Under conditions of production uncertainty, but with perfect knowledge, the rate would differ from year to year, depending upon the productivity of nitrogen.

In actuality, however, the corn producer is not faced with perfect knowledge. Hence, he must make plans in the face of uncertainty. The decision as to how much nitrogen to apply under uncertainty may be based upon knowledge of some past period. If plans are based on responses obtained in some past period, procedures such as basing plans upon last year's function, a parallel period, best responses, poorest responses, mean responses, median responses, first quartile responses, or third quartile responses are a few of the planning procedures which may be used. For the first comparison of these systems of planning the mean price of corn during the harvest seasons of the 32-year period and the mean price of nitrogen in the form of Nitrate of Soda were used. These prices were $1.06 per bushel for corn and $1.17 per $7\frac{1}{2}$ pound increment for nitrogen, or a price ratio of 1.10. Prices used for the second comparison were $1.06 per bushel for corn and $3.50 per $7\frac{1}{2}$ pound increment for nitrogen or a price ratio of 3.30.

The indicated most profitable rate of nitrogen, if its application were profitable at all, was an infinite quantity for linear functions. The same was true for functions having a range of increasing marginal productivity not followed by a range of decreasing marginal productivity. In these cases, and in all other cases where the indicated most profitable rate of application was greater than the highest rate used in the experiment, six increments of $7\frac{1}{2}$ pounds was used as the most profitable rate.

The various systems of planning, except for planning in terms of the poorest responses, would have yielded similar results under the price ratio 1.10. This was due, primarily, to the use of six increments as the most profitable rate of nitrogen application when a greater rate was indicated. Under the price ratio 3.30, however, the rate used as the most profitable rate was more dependent upon the planning procedure followed. Hence, greater differences in results which would have been obtained were shown.

Under the latter price ratio no nitrogen would ever have been applied if plans were based on poorest responses. Consequently, neither profits nor losses would have been realized. Average profit from the application of nitrogen would have been greatest if plans had been based on mean or median responses and smallest if the parallel period system were used. Planning in terms of best responses resulted in losses the greatest number of years and plans based on third quartile responses resulted in losses the smallest number of years. In addition, maximum loss was smallest and percentage return on money invested in nitrogen was greatest for the latter planning procedure.

It appears that plans based on third quartile responses would yield satisfactory results for the farmer with limited capital and alternative investment opportunities. For the farmer with unlimited capital, however, the procedure whereby plans were based on mean or median responses would seem to be superior. Losses in any particular year would be of no concern since maximization of profits over a period of years would be the objective.

APPENDIX

Table A. Corrected sums of squares and cross products for corn-nitrogen data,[a] Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Sy^2	Sx_1y	Sx_2y	Sx_3y	Sx_4y
1921	22.689	20.1	97.5	459.3	8.494
1922	539.889	121.7	735.5	4,146.5	44.209
1923	334.477	93.4	554.4	3,115.6	34.567
1924	258.554	81.8	460.0	2,476.4	31.662
1925	24.574	8.4	10.6	-53.4	5.674
1926	389.829	102.0	582.4	3,166.2	39.118
1927	1,022.789	161.4	903.8	4,833.6	62.538
1928	1,630.609	210.7	1,219.7	6,711.1	80.262
1929	439.989	108.4	633.2	3,488.2	40.256
1930	246.454	81.7	479.5	2,643.7	30.296
1931	241.269	80.4	455.2	2,478.6	31.168
1932	685.969	135.8	814.4	4,549.4	49.389
1933	181.497	68.6	439.8	2,559.2	23.342
1934	280.117	86.4	494.0	2,683.2	32.836
1935	662.794	134.0	810.4	4,551.2	48.359
1936	107.500	53.3	313.5	1,735.1	20.059
1937	565.569	122.8	701.0	3,817.0	47.103
1938	846.449	150.7	943.3	5,437.3	53.140
1939	317.709	93.9	553.9	3,090.3	35.232
1940	329.080	95.4	573.4	3,230.4	34.880
1941	294.017	89.3	526.9	2,966.3	33.527
1942	906.114	158.3	925.7	5,158.1	59.961
1943	431.909	107.5	614.7	3,384.1	41.553
1944	94.814	43.3	232.3	1,170.1	16.673
1945	1,010.717	166.5	972.9	5,411.7	62.984
1946	1,082.917	171.5	1,043.5	5,888.3	61.435
1947	654.337	135.1	819.5	4,663.3	49.548
1948	594.117	127.4	766.0	4,313.0	46.502
1949	1,282.894	188.7	1,139.9	6,440.7	68.912
1950	1,134.394	177.2	1,064.2	6,009.8	65.146
1951	412.874	105.5	656.7	3,811.1	37.674
1952	792.694	147.9	896.1	5,078.1	54.112

[a] Y denotes corn yield in bushels per acre and X_1 nitrogen application per acre in $7\frac{1}{2}$ pound increments. $X_2 = X_1^2$; $X_3 = X_1^3$; and $X_4 = X_1^{\frac{1}{2}}$. $Sx_1^2 = 28$, $Sx_2^2 = 1{,}092$, $Sx_3^2 = 39{,}388$, $Sx_4^2 = 4.2378$, $Sx_1x_2 = 168$, $Sx_1x_4 = 10.405$, $Sx_2x_3 = 6{,}468$, and $Sx_2x_4 = 57.505$.

Table B. Marginal productivity[a] of nitrogen at various levels, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	Level of nitrogen in 7½ pound increments					
	1	2	3	4	5	6
			(bushels)			
1921	1.2	2.2	1.4	.6	-.8	-2.9
1922	3.1	5.0	5.7	5.2	3.7	.9
1923	3.3	3.3	3.3	3.3	3.3	3.3
1924	2.9	4.3	4.5	3.2	.6	-3.5
1925	1.7	.6	-.1	-.6	-1.2	-1.7
1926	3.4	5.2	5.4	4.1	1.2	-3.3
1927	6.0	8.9	9.1	6.3	.6	-7.9
1928	9.6	17.2	22.6	25.8	27.0	26.1
1929	3.3	5.2	5.5	4.5	1.9	-2.0
1930	2.5	3.9	4.2	3.4	1.5	-1.5
1931	4.2	3.5	2.9	2.2	1.6	.9
1932	3.7	5.9	6.6	5.8	3.4	-.5
1933	1.2	2.1	2.8	3.2	3.3	3.2
1934	2.9	4.4	4.6	3.5	1.0	-2.9
1935	3.5	5.6	6.5	5.8	3.8	.4
1936	1.6	2.4	2.6	2.3	1.1	-.7
1937	4.1	6.2	6.5	5.0	1.5	-3.8
1938	3.0	5.1	6.3	6.8	6.5	5.3
1939	3.8	3.5	3.3	3.1	2.9	2.6
1940	2.4	3.9	4.4	4.1	2.8	.6
1941	3.2	3.2	3.2	3.2	3.2	3.2
1942	6.8	6.3	5.7	5.1	4.5	4.0
1943	5.3	4.6	3.8	3.1	2.4	1.7
1944	2.1	3.1	2.9	1.6	-.9	-4.5
1945	7.2	6.6	5.9	5.3	4.7	4.1
1946	4.3	7.0	8.1	7.5	5.2	1.5
1947	4.6	4.9	5.0	5.1	5.1	5.1
1948	3.3	5.2	6.0	5.5	3.7	.7
1949	4.6	7.5	8.7	8.1	5.9	1.8
1950	6.3	6.3	6.3	6.3	6.3	6.3
1951	2.7	3.2	3.8	4.4	4.9	5.5
1952	3.6	5.7	6.7	6.5	4.8	2.0

[a]Calculated as $\frac{dy}{dx}$ at the levels specified.

Table C. Rainfall and temperature June-August, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952[a]

Year or average	Rainfall June	Rainfall July	Rainfall August	Temperature June	Temperature July	Temperature August
		(inches)			(degrees Fahrenheit)	
1921	2.6	3.7	1.8	80.8	83.2	83.8
1922	5.2	4.2	1.8	80.0	81.8	81.6
1923	4.6	4.9	7.1	79.0	80.6	81.5
1924	.3	.8	.8	82.2	83.2	84.2
1925	1.8	2.2	2.3	83.2	83.6	82.2
1926	1.1	1.9	3.4	79.3	81.6	83.3
1927	4.0	1.7	4.5	78.0	83.4	80.2
1928	4.7	6.1	4.8	78.4	83.0	84.4
1929	2.0	2.1	2.1	79.8	82.9	82.0
1930	.0	.5	1.3	77.4	85.4	81.6
1931	.5	4.8	2.2	79.0	80.2	77.1
1932	6.3	6.6	1.3	81.5	83.3	82.2
1933	1.3	6.0	2.4	77.9	81.1	80.2
1934	6.4	3.4	.4	80.0	82.2	82.3
1935	5.8	1.8	1.5	77.4	82.2	82.6
1936	1.1	4.2	1.0	80.6	82.2	83.4
1937	4.2	1.6	3.7	81.9	82.3	80.6
1938	5.0	3.9	5.7	76.1	81.6	82.4
1939	5.6	2.0	.5	79.7	82.5	83.0
1940	2.7	13.2	2.8	77.2	79.0	80.2
1941	2.4	3.8	2.5	79.8	82.2	82.5
1942	6.0	1.5	4.9	79.4	81.8	80.0

[a]U. S. Department of Commerce. Weather Bureau. Climatological data.

Table C. (Continued)

Year or average	Rainfall June	Rainfall July	Rainfall August	Temperature June	Temperature July	Temperature August
		(inches)			(degrees Fahrenheit)	
1943	2.6	1.1	4.0	81.8	82.7	83.8
1944	1.6	1.6	5.4	81.9	82.7	81.7
1945	3.5	7.4	2.3	77.9	80.0	80.2
1946	7.0	3.8	2.3	76.7	80.2	79.5
1947	5.8	2.1	.9	78.4	78.4	83.2
1948	4.0	2.4	4.6	80.4	83.4	79.4
1949	3.8	5.4	1.7	79.8	82.1	80.7
1950	3.0	7.3	4.4	78.1	78.7	77.7
1951	6.2	6.1	.1	79.0	81.9	82.9
1952	1.4	1.8	1.2	83.8	83.7	82.5
			Mean			
12 yrs.[b]	2.84	2.52	2.33	80.6	83.0	82.3
20 yrs.	3.92	4.48	2.88	79.0	81.4	81.3

[b]The twelve years for which negative marginal productivity is indicated at or before six increments of nitrogen (1921, 1924-1927, 1929, 1930, 1932, 1934, 1936, 1937, and 1944).

Table D. Comparison of regression coefficients of linear production functions[a] for the corn-nitrogen data, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Year	d.f	Sx^2	Sxy	Sy^2	$Sy^2 - \frac{(Sxy)^2}{Sx^2}$	d.f	M.S.
1923	6	28	93.4	334.477	22.921	5	
1941	6	28	89.3	294.017	9.214	5	
1950	6	28	177.2	1,134.394	12.971	5	
					45.106	15	3.007
Totals	18	84	359.9	1,762.888	220.888	17	
Difference for testing $H_o: \beta_1 = \beta_2 = \ldots = \beta_n$					175.782	2	87.891

$F = 87.891/3.007 = 29.23^{**}$

[a]For the years for which a linear function was selected to describe the relationships between corn yields and nitrogen applications. Functions are listed in Table 3.

Table E. Determination of weights used in fitting ~~selected~~ planning functions, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952

Rates of N[a]	Sy^2	$S^2 = Sy^2/31$	Weight $= 10{,}000/S^2$
1	154.18	4.97	2,012
2	418.12	13.49	741
3	819.64	26.44	378
4	1,415.40	45.66	219
5	2,168.52	69.95	143
6	3,091.78	99.73	100

[a]Rates of nitrogen in $7\frac{1}{2}$ pound increments per acre. Y denotes the response to nitrogen in bushels per acre as determined from the annual functions.

Table F. Weighted sums of squares and cross products for calculation of mean, median, first quartile, and third quartile response functions, Delta Branch Experiment Station, Stoneville, Mississippi, 1921-1952[a]

Type of function	Sy^2	Sx_1y	Sx_2y	Sx_3y	Sx_4y
Mean	116,808.33	26,636.75	152,200.85	774,878.14	8,498.51
Median	125,368.36	27,593.60	157,862.22	804,289.66	8,797.38
First quartile	73,804.59	21,138.58	120,240.05	609,535.96	6,759.17
Third quartile	167,408.38	31,952.18	185,226.35	955,367.75	10,122.16

[a]Y denotes response to nitrogen in bushels per acre as determined from the annual functions. X_1 denotes nitrogen application per acre in $7\frac{1}{2}$ pound increments. $X_2 = X_1^2$; $X_3 = X_1^3$; and $X_4 = X_1^{\frac{1}{2}}$.
$Sx_1^2 = 6,115.52$; $Sx_2^2 = 218,448.12$; $Sx_3^2 = 6,693,703.16$; $Sx_4^2 = 626.07$;
$Sx_1x_2 = 35,469.55$; $Sx_1x_3 = 185,568.23$; $Sx_1x_4 = 1,939.37$;
$Sx_2x_3 = 1,186,351.78$; and $Sx_2x_4 = 10,894.22$.